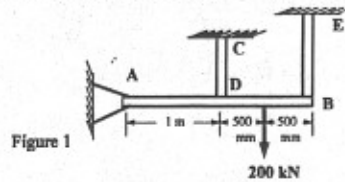


2004 GE 213.3 Solutions Midterm Exam

1. The bar AB is considered to be absolutely rigid and is horizontal before the load of 200 kN is applied, as shown in Fig. 1. The connection at A is a pin, and AB is supported by the steel rod EB , and the copper rod CD . The length of CD is 0.75 m, and of EB is 1.5 m. The cross-sectional area of CD is 400 mm², and the area of EB is 200 mm². Determine the stress in each of the vertical rods and the elongation of steel rod EB [$\delta = PL/AE$]. Neglect the weight of AB . For copper, $E=120$ GPa; for steel, $E= 200$ GPa.



SOLUTION:

(a) Given:

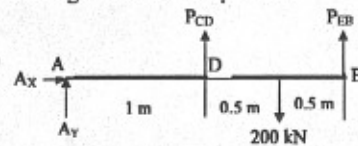
Rod CD: Copper $E= 120$ GPa; Length = 0.75m and Area = 400 mm²
 Rod EB: Steel $E = 200$ GPa; Length = 1.5 m and Area = 200 mm²

Now the Free-Body Diagram of ADB and taking Moment about point A

$$+\circlearrowleft \sum M_A = 0$$

$$1.0 P_{CD} + 2.0 P_{EB} - 1.5 (200 \text{ kN}) = 0$$

$$\text{or } P_{CD} + 2 P_{EB} = 300 \text{ kN} \quad \text{---(1)}$$

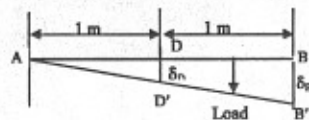


For another relation, we draw the deformation diagram (indeterminate problem)

From similar triangles ADD' and ABB'

$$\frac{DD'}{AD} = \frac{BB'}{AB} \quad \text{or} \quad \frac{\delta_D}{1} = \frac{\delta_B}{2}$$

$$\text{or} \quad 2\delta_D = \delta_B \quad \text{---(2)}$$



Now, deflection, $\delta = \frac{PL}{AE}$

Therefore,

$$\delta_D = \frac{P_{CD} L_{CD}}{A_{CD} E_{CD}} = \frac{P_{CD} (0.75 \text{ m})}{(400 \text{ mm}^2 \times 10^{-6}) (120 \times 10^9 \text{ Pa})} = 15.625 \times 10^{-9} P_{CD}$$

$$\text{Similarly, } \delta_B = \frac{P_{EB} L_{EB}}{A_{EB} E_{EB}} = \frac{P_{EB} (1.5 \text{ m})}{(200 \text{ mm}^2 \times 10^{-6}) (200 \times 10^9 \text{ Pa})} = 37.5 \times 10^{-9} P_{EB}$$

Substituting in Eq. (2): $2\delta_D = \delta_B$

$$2(15.625 \times 10^{-9}) P_{CD} = (37.5 \times 10^{-9}) P_{EB}$$

$$\text{or} \quad P_{EB} = 5/6 P_{CD}$$

Substituting this in Eq. (1): $P_{CD} + 2 P_{EB} = 300 \text{ kN}$, we get

$$P_{CD} + 2(5/6) P_{CD} = 300 \text{ kN} \quad \text{or} \quad P_{CD} = (3/8) 300 \text{ kN} = 112.5 \text{ kN}$$

$$\text{and } P_{EB} = 5/6 P_{CD} = 93.75 \text{ kN}$$

Now Stresses in rods

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{112.5 \text{ kN}}{400 \text{ mm}^2 \times 10^{-6}} = 281.25 \text{ MPa} \quad \leftarrow \text{Ans.}$$

$$\sigma_{EB} = \frac{P_{EB}}{A_{EB}} = \frac{93.75 \text{ kN}}{200 \text{ mm}^2 \times 10^{-6}} = 468.75 \text{ MPa} \quad \leftarrow \text{Ans.}$$

Elongation of steel rod: $\delta_{\text{steel}} = \delta_B$

$$\delta_B = \frac{P_{EB} L_{EB}}{A_{EB} E_{EB}} = \frac{P_{EB} (1.5 \text{ m})}{(200 \text{ mm}^2 \times 10^{-6}) (200 \times 10^9 \text{ Pa})} = 37.5 \times 10^{-9} P_{EB}$$

$$\delta_B = (37.5 \times 10^{-9}) (93.75 \times 10^3) = 3.516 \times 10^{-3} \text{ m} = 3.516 \text{ mm} \quad \leftarrow \text{Ans.}$$

2. The two steel shafts shown in Figure 2 are coupled together using the meshed gears. The shaft AB is free to rotate within bearings, whereas, shaft CD is fixed at D. The specifications of gears and shafts are given in the Figure. For each shaft the value of $G = 80$ GPa, and the allowable shearing stress is 60 MPa. Determine
 (a) the largest torque T_0 that may be applied to end A of shaft AB, and
 (b) the corresponding angle of twist of end A of the shaft AB.

Note: Based on Example Solution 3.4, page 156

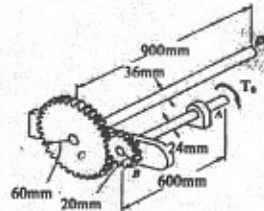


Figure 2

Solution:

Determine the Torque on shaft CD

Relation between torques:

$$\frac{T_{CD}}{T_0} = \frac{r_C}{r_B} \quad \therefore T_{CD} = T_0 \cdot \frac{r_C}{r_B} = T_0 \times \frac{60\text{mm}}{20\text{mm}} = 3T_0 \quad (1)$$

Relation between angles of rotation of gears: since the length of arc of both gears are equal

$$r_C \cdot \phi_C = r_B \phi_B \quad \therefore \phi_B = \phi_C \cdot \frac{r_C}{r_B} = \frac{60\text{mm}}{20\text{mm}} \phi_C = 3\phi_C \quad (2)$$

To determine the Torque T_0

Shaft AB: $T_{CD} = 3T_0$, and $c = 18$ mm with maximum allowable shearing stress of 60 MPa

$$\tau = \frac{T_0 c}{J} \text{ or } T_0 = \frac{\tau J}{c} = \frac{(60 \times 10^6 \text{ Pa}) \left(\frac{\pi}{2} (18)^4 \times 10^{-12} \text{ m}^4 \right)}{12 \times 10^{-3} \text{ m}} = 162.86 \text{ N.m} \quad [J = 32.572 \times 10^9 \text{ m}^4]$$

Shaft CD: $T_{AB} = T_0$, and $c = 12$ mm with maximum allowable shearing stress of 60 MPa

$$T_0 = \frac{\tau J}{3c} = \frac{(60 \times 10^6 \text{ Pa}) \left(\frac{\pi}{2} (12)^4 \times 10^{-12} \text{ m}^4 \right)}{3(18 \times 10^{-3} \text{ m})} = 183.22 \text{ N.m} \quad [J = 164.896 \times 10^9 \text{ m}^4]$$

Therefore, permissible torque $T_0 = 162.86 \text{ N.m} \approx 163 \text{ N.m} \quad \leftarrow \text{ANS.}$

Now, angle of rotation of End A:

First the angle of twist of each shaft

$$\phi_{A/B} = \frac{T_0 \cdot L_{AB}}{JG} = \frac{(162.86 \text{ N.m})(0.6\text{m})}{[32.572 \times 10^{-9} \text{ m}^4](80 \times 10^9 \text{ Pa})} = 0.0375 \text{ rad or } 2.15 \text{ deg.}$$

$$\phi_{C/D} = \frac{3T_0 \cdot L_{CD}}{JG} = \frac{3(162.86 \text{ N.m})(0.9\text{m})}{[164.896 \times 10^{-9} \text{ m}^4](80 \times 10^9 \text{ Pa})} = 0.0333 \text{ rad or } 1.91 \text{ deg.}$$

The rotation of gear C (ϕ_C) will be equal to rotation of shaft CD.

Therefore, rotation of gear B caused by the rotation of gear C (Eq. 2)

$$\phi_B = 3\phi_C = 3(1.91 \text{ deg}) = 5.72 \text{ deg or } 3(0.0333 \text{ rad}) = 0.1 \text{ rad}$$

Therefore, rotation of the End A of Shaft AB: $\phi_A = \phi_B + \phi_{A/B}$

$$= 0.1 \text{ rad} + 0.0375 \text{ rad} = 0.1375 \text{ rad}$$

$$\text{OR } = 5.72 \text{ deg} + 2.15 \text{ deg} = 7.87 \text{ deg} \quad \leftarrow \text{ANS.}$$

3. A 400 mm long, 16 mm diameter rod made of homogenous isotropic material is observed to increase in length by 250 μm , and to decrease in diameter by 2.4 μm when subjected to an axial load of 12 kN. Determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material. $[G = E/2(1+\nu)]$

Note: Similar to solved Example 2.07, page 85 from TEXT

Solution:

$$L = 400 \text{ mm} \quad \delta_x = 250 \mu\text{m}$$

Cross-sectional area of rod:

$$A = \pi r^2 = \pi (8 \times 10^{-3} \text{ m})^2 \\ = 201 \times 10^{-6} \text{ m}^2$$

Choosing x-axis along the axis of the rod

$$\sigma_x = \frac{P}{A} = \frac{12 \times 10^3 \text{ N}}{201 \times 10^{-6} \text{ m}^2} = 59.7 \text{ MPa} \quad \text{Fig. 2.41 (from Text page 85)}$$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{250 \times 10^{-6} \text{ m}}{400 \times 10^{-3} \text{ m}} = 625 \times 10^{-6} \quad \text{and} \quad \epsilon_y = \frac{\delta_y}{d} = \frac{-2.4 \times 10^{-6} \text{ m}}{16 \times 10^{-3} \text{ m}} = -150 \times 10^{-6}$$

Using Hooke's law, $\sigma_x = E\epsilon_x$:

$$E = \frac{\sigma_x}{\epsilon_x} = \frac{59.7 \times 10^6 \text{ Pa}}{625 \times 10^{-6}} = 95.52 \text{ GPa} \leftarrow \text{Ans.}$$

Poisson's ratio = [Lateral strain/ Axial strain]

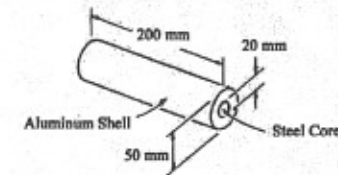
$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{-150 \times 10^{-6}}{625 \times 10^{-6}} = 0.24 \leftarrow \text{Ans.}$$

Using the relation $G = E/[2(1+\nu)]$:

$$G = \frac{E}{2(1+\nu)} = \frac{95.52 \times 10^9 \text{ Pa}}{2(1+0.24)} = 38.52 \text{ GPa} \leftarrow \text{Ans.}$$

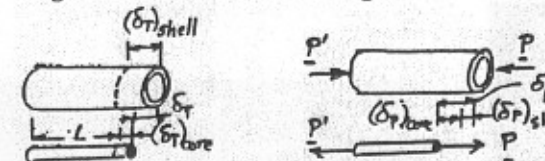
4. The assembly in Fig. 3 consists of an aluminum shell ($E_s = 70 \text{ GPa}$; $\alpha_s = 23.6 \times 10^{-6}/^\circ\text{C}$) fully bonded to a steel core ($E_c = 200 \text{ GPa}$; $\alpha_c = 11.7 \times 10^{-6}/^\circ\text{C}$) and is unstressed at a temperature of 20°C . Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180°C .

Figure 3



Solution:

Consider the aluminum shell and steel core first expand and then use a force to bring the two elements to the same length.



If the aluminum shell and the steel core were not bonded, they would expand through

$$(\delta_T)_{sh} = \alpha_{sh} \Delta T L, \quad (\delta_T)_{core} = \alpha_{core} \Delta T L$$

Thus, due to temperature, the shell would be longer by

$$\delta_T = (\delta_T)_{sh} - (\delta_T)_{core} = (\alpha_{sh} - \alpha_{core}) \Delta T L \quad (1)$$

$$\text{Substituting given data: } \alpha_{sh} = \alpha_s = 23.6 \times 10^{-6}, \quad \alpha_{core} = \alpha_c = 11.7 \times 10^{-6} \\ \Delta T = 180^\circ\text{C} - 20^\circ\text{C} = 160^\circ\text{C}, \quad L = 0.2 \text{ m}$$

$$\text{From Eq. (1)} \quad \delta_T = (23.6 - 11.7)10^{-6}(160^\circ)(0.2 \text{ m}) = 380.8 \times 10^{-6} \text{ m}$$

Now the Force applied to the assembly:

$$\delta_T = (\delta_T)_{sh} + (\delta_T)_{core} = \frac{PL}{A_{sh}E_{sh}} + \frac{PL}{A_{core}E_{core}} \quad (2)$$

$$A_{sh} = \frac{\pi}{4} (0.05^2 - 0.02^2) = 1.6493 \times 10^{-3} \text{ m}^2 \quad E_{sh} = E_s = 70 \times 10^9 \text{ Pa}$$

$$A_{core} = \frac{\pi}{4}(0.02^2) = 0.31416 \times 10^{-3} m^2 \quad E_{core} = E_s = 200 \times 10^9 Pa$$

$$\begin{aligned} \text{From Eq. (2), } \delta_p &= \frac{P(0.2)}{1.6493 \times 10 \times 10^6} + \frac{P(0.2)}{0.31416 \times 200 \times 10^9} \\ &= 1.7323 \times 10^{-9} P + 3.1831 \times 10^{-9} P \\ &= 4.9154 \times 10^{-9} P \end{aligned}$$

Since shell and core must have same final length, we set

$$\delta_p = \delta_r \text{ or } 4.9154 \times 10^{-9} P = 380.8 \times 10^{-6}$$

$$\therefore P = 77.471 \text{ kN}$$

Now the stress in the shell

$$\sigma_{shell} = \frac{P}{A_{sh}} = \frac{77.471 \times 10^3 N}{1.6493 \times 10^{-3} m^2} = -46.972 \times 10^6 Pa$$

$$\sigma_{shell} = -47.0 \text{ MPa} \quad \leftarrow \text{ANS.}$$
