2004 GE 213.3 Solutions Midterm Exam

1. The bar *AB* is considered to be absolutely rigid and is horizontal before the load of 200 kN is applied, as shown in Fig. 1. The connection at *A* is a pin, and *AB* is supported by the steel rod *EB*, and the copper rod *CD*. The length of *CD* is 0.75 m, and of *EB* is 1.5 m. The cross-sectional area of *CD* is 400 mm², and the area of *EB* is 200 mm². Determine the stress in each of the vertical rods and the elongation of steel rod *EB* [$\delta = PL/AE$]. Neglect the weight of *AB*. For copper, *E*=120 GPa; for steel, *E*= 200 GPa.

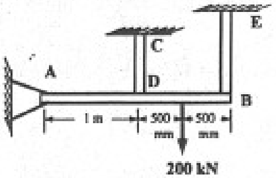


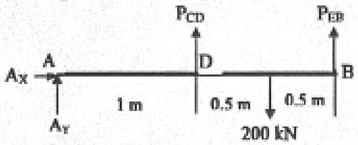
Figure 1

SOLUTION:

(a) Given:

Rod CD: Copper E= 120 GPa; Length = 0.75m and Area = 400 mm²
Rod EB: Steel E = 200 GPa; Length = 1.5 m and Area = 200 mm²

Now the Free-Body Diagram of ADB and taking Moment about point A



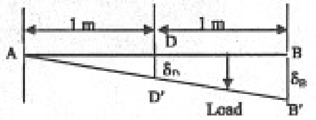
$+\circlearrowleft\Sigma M_A = 0$

$1.0\ P_{CD} + 2.0\ P_{EB} - 1.5\ (200\ kN) = 0$

or $P_{CD} + 2\ P_{EB} = 300\ kN$ ------------(1)

For another relation, we draw the deformation diagram (inderminate problem)

From similar triangles ADD′ and ABB′



$$\frac{DD'}{AD} = \frac{BB'}{AB} \quad or \quad \frac{\delta_D}{1} = \frac{\delta_B}{2}$$

or $2\delta_D = \delta_B$ ------------(2)

Now, deflection, $\delta = \dfrac{PL}{AE}$

Therefore,

$$\delta_D = \frac{P_{CD}\,L_{CD}}{A_{CD}\,E_{CD}} = \frac{P_{CD}(0.75m)}{(400\,mm^2\times10^{-6})(120\times10^9\,Pa)} = 15.625\times10^{-9}\,P_{CD}$$

Similarly, $\delta_B = \dfrac{P_{EB}\,L_{EB}}{A_{EB}\,E_{EB}} = \dfrac{P_{EB}(1.5m)}{(200\,mm^2\times10^{-6})(200\times10^9\,Pa)} = 37.5\times10^{-9}\,P_{EB}$

Substituting in Eq. (2): $2\delta_D = \delta_B$

$2(15.625\text{x}10^{-9})\ P_{CD} = (37.5\ \text{x}\ 10^{-9})P_{EB}$

or $P_{EB} = 5/6\ P_{CD}$

Substituting this in Eq. (1): $P_{CD} + 2\ P_{EB} = 300\ kN$, we get

$P_{CD} + 2(5/6)\ P_{CD} = 300\ kN$ *or* $P_{CD} = (3/8)\ 300\ kN = 112.5\ kN$

and $P_{EB} = 5/6\ P_{CD} = 93.75\ kN$

Now Stresses in rods

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{112.5\,kN}{400\,mm^2\times10^{-6}} = 281.25\,MPa \quad \Leftarrow \textbf{Ans.}$$

$$\sigma_{EB} = \frac{P_{EB}}{A_{EB}} = \frac{93.751\,kN}{200\,mm^2\times10^{-6}} = 468.75\,MPa \quad \Leftarrow \textbf{Ans.}$$

Elongation of steel rod: $\delta_{steel} = \delta_B$

$$\delta_B = \frac{P_{EB}\,L_{EB}}{A_{EB}\,E_{EB}} = \frac{P_{EB}(1.5m)}{(200\,mm^2\times10^{-6})(200\times10^9\,Pa)} = 37.5\times10^{-9}\,P_{EB}$$

$$\delta_B = (37.5\times10^{-9})(93.75\times10^3) = 3.516\times10^{-3}\,m = 3.516\,mm \quad \Leftarrow \textbf{Ans.}$$

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2. The two steel shafts shown in Figure 2 are coupled together using the meshed gears. The shaft AB is free to rotate within bearings, whereas, shaft CD is fixed at D. The specifications of gears and shafts are given in the Figure. For each shaft the value of G = 80 GPa, and the allowable shearing stress is 60 MPa. Determine
   (a) the largest torque T_0 that may be applied to end A of shaft AB, and
   (b) the corresponding angle of twist of end A of the shaft AB.

Note: Based on Example Solution 3.4, page 156

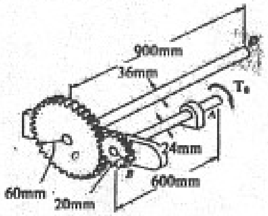


Figure 2

Solution:

Determine the Torque on shaft CD

Relation between torques:

$$\frac{T_{CD}}{T_0}=\frac{r_C}{r_B} \quad \therefore T_{CD}=T_0\cdot\frac{r_C}{r_B}=T_0\times\frac{60mm}{20mm}=3T_0 \qquad (1)$$

Relation between angles of rotation of gears: since the length of arc of both gears are equal

$$r_C\cdot\phi_C=r_B\phi_B \quad \therefore \phi_B=\phi_C\cdot\frac{r_C}{r_B}=\frac{60mm}{20mm}\phi_C=3\phi_C \qquad (2)$$

To determine the Toque T_0

Shaft AB: $T_{CD}=3T_0$, and c = 18 mm with maximum allowable shearing stress of 60 MPa

$$\tau=\frac{T_0 c}{J} \text{ or } T_0=\frac{\tau J}{c}=\frac{(60\times10^6 Pa)(\frac{\pi}{2}(12)^4\times10^{-12}m^4)}{12\times10^{-3}m}=162.86 N.m \quad [J=32.572\times10^{-9}m^4]$$

Shaft CD: $T_{AB}=T_0$, and c = 12 mm with maximum allowable shearing stress of 60 MPa

$$T_0=\frac{\tau J}{3c}=\frac{(60\times10^6 Pa)(\frac{\pi}{2}(18)^4\times10^{-12}m^4)}{3(18\times10^{-3}m)}=183.22 N.m \qquad [J=164.896\times10^9 m^4]$$

Therefore, permissible torque <u>T_0 = 162.86 N.m ≈ 163 N.m</u> ⇐*ANS.*

Now, angle of rotation of End A:

First the angle of twist of each shaft

$$\phi_{A/B}=\frac{T_0\cdot L_{AB}}{JG}=\frac{(162.86N.m)(0.6m)}{[32.572\times10^{-9}m^4](80\times10^9 Pa)}=0.0375 rad \text{ or } 2.15 deg.$$

$$\phi_{C/D}=\frac{3T_0\cdot L_{CD}}{JG}=\frac{3(162.86N.m)(0.9m)}{[164.896\times10^{-9}m^4](80\times10^9 Pa)}=0.0333 rad \text{ or } 1.91 deg.$$

The rotation of gear C (ϕ_C) will be equal to rotation of shaft CD.

Therefore, rotation of gear B caused by the rotation of gear C (Eq. 2)

$$\phi_B=3\phi_C=3(1.91 deg)=5.72 deg \text{ or } 3(0.0333 rad)=0.1 rad$$

Therefore, rotation of the End A of Shaft AB: $\phi_A=\phi_B+\phi_{A/B}$

= 0.1 rad + 0.0375 rad = <u>0.1375 rad</u>

OR = 5.72 deg + 2.15 deg = <u>7.87deg</u> ⇐*ANS.*

3. A 400 mm long, 16 mm diameter rod made of homogenous isotropic material is observed to increase in length by 250 μm, and to decrease in diameter by 2.4 μm when subjected to an axial load of 12 kN. Determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material. *[G=E/2(1+ν)]*

Note: Similar to solved Example 2.07, page 85 from TEXT

Solution:

L=400 mm δ_x=250μm

Cross-sectional area of rod:

$A = \pi r^2 = \pi\,(8 \times 10^{-3}\ \text{m})^2$

$= 201 \times 10^{-6}\ \text{m}^2$

Choosing x-axis along the axis of the rod

$$\sigma_x = \frac{P}{A} = \frac{12\times10^3\,N}{201\times10^{-6}\,m^2} = 59.7\ MPa$$

Fig. 2.41 (from Text page 85)

$$\varepsilon_x = \frac{\delta_x}{L} = \frac{250\times10^{-6}\,m}{400\times10^{-3}\,m} = 625\times10^{-6} \quad \text{and} \quad \varepsilon_y = \frac{\delta_y}{d} = \frac{-2.4\times10^{-6}\,m}{16\times10^{-3}\,m} = -150\times10^{-6}$$

Using Hooke's law, $\sigma_x = E\varepsilon_x$:

$$E = \frac{\sigma_x}{\varepsilon_x} = \frac{59.7\times10^6\,Pa}{625\times10^{-6}} = 95.52\ GPa \Leftarrow Ans.$$

Poisson's ratio = |Lateral strain/ Axial strain|

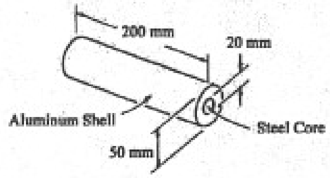
$$\upsilon = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{-150\times10^{-6}}{625\times10^{-6}} = 0.24 \Leftarrow Ans.$$

Using the relation G=E/[2(1+υ)]:

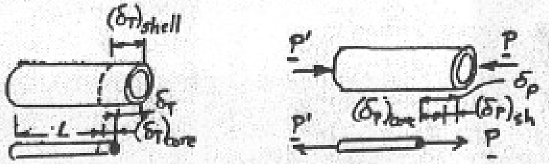
$$G = \frac{E}{2(1+\upsilon)} = \frac{95.52\times10^9\,Pa}{2(1+0.24)} = 38.52\ GPa \Leftarrow Ans.$$

4. The assembly in Fig. 3 consists of an aluminum shell (E_a = 70 GPa; α_a = 23.6 x 10^{-6}/°C) fully bonded to a steel core (E_s = 200 GPa; α_s = 11.7 x 10^{-6}/°C) and is unstressed at a temperature of 20°C. Considering only axial deformations, determine the stress in the aluminum shell when the temperature reaches 180°C.

Figure 3



Solution:
Consider the aluminum shell and steel core first expand and then use a force to bring the two elements to the same length.



If the aluminum shell and the steel core were not bonded, they would expand through

$(\delta_T)_{sh} = \alpha_{sh}\Delta TL, \qquad (\delta_T)_{core} = \alpha_{core}\Delta TL$

Thus, due to temperature, the shell would be longer by

$$\delta_T = (\delta_T)_{sh} - (\delta_T)_{core} = (\alpha_{sh} - \alpha_{core})\Delta TL \qquad (1)$$

Substituting given data: $\alpha_{sh} = \alpha_a = 23.6x10^{-6}, \quad \alpha_{core} = \alpha_s = 11.7x10^{-6}$

$\Delta T = 180°C - 20°C = 160°C, \quad L = 0.2m$

From Eq. (1) $\delta_T = (23.6 - 11.7)10-6(160°)(0.2m) = 380.8x10^{-6}\,m$

Now the Force applied to the assembly:

$$\delta_P = (\delta_P)_{sh} + (\delta_P)_{core} = \frac{PL}{A_{sh}E_{sh}} + \frac{PL}{A_{core}E_{core}} \qquad (2)$$

$A_{sh} = \frac{\pi}{4}(0.05^2 - 0.02^2) = 1.6493x10^{-3}\,m^2 \qquad E_{sh} = E_a = 70x10^9\,Pa$

$A_{core} = \frac{\pi}{4}(0.02^2) = 0.31416x10^{-3} m^2 \qquad E_{core} = E_s = 200x10^9\ Pa$

From Eq. (2), $\delta_P = \frac{P(0.2)}{1.6493x10x10^6} + \frac{P(0.2)}{0.31416x200x10^6}$

$= 1.7323x10^{-9} P + 3.1831x10^{-9} P$

$= 4.9154x10^{-9} P$

Since shell and core must have some final length, we set

$\delta_P = \delta_T$ or $4.9154x10^{-9} P = 380.8x10^{-6}$

∴ P=77.471 kN

Now the stress in the shell

$$\sigma_{shell} = -\frac{P}{A_{sh}} = -\frac{77.471x10^3\ N}{1.6493x10^{-3} m^2} = -46.972x10^6\ Pa$$

$\sigma_{shell} = -47.0\ MPa$ ⇐*ANS.*