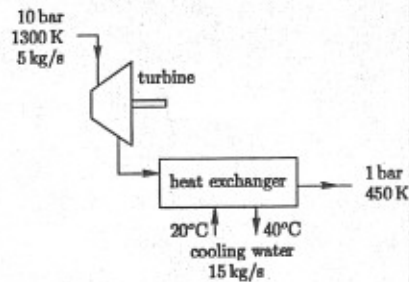


M E 227.3 Thermodynamics I
Department of Mechanical Engineering
Midterm Examination
October 27, 2004

Time: 90 minutes
Calculators Allowed
Formula Sheet Supplied

Total Marks: 50
Closed Book
J.D. Bugg

- (10) 1. Define the following terms.
- Enthalpy
 - Total Energy
 - Ideal Gas
 - Saturated Vapour
 - Isobaric Process
- (20) 2. A piston-cylinder device contains 2 kg of water which is initially a saturated liquid at 300°C. It undergoes three processes to form a cycle. The first process is constant volume, the second is constant pressure at 2 bar and the third is a polytropic process where $Pv^{1.1} = \text{constant}$. Calculate heat transfer and work (both in kJ) for each process. Is the device a heat engine or a refrigerator?
- (20) 3. Air ($R = 286.9 \text{ J}/(\text{kg} \cdot \text{K})$) at 1300 K and 10 bar enters a turbine with a mass flowrate of 5 kg/s. After exiting the turbine, the air enters a heat exchanger where it is cooled to 450 K before discharging to the atmosphere at 1 bar. The heat exchanger warms liquid water flowing at 15 kg/s from 20°C to 40°C. Calculate the power output of the turbine (MW). For the cooling water, you may assume that $c_p = 4.18 \text{ kJ}/(\text{kg} \cdot \text{K})$ is a constant.



Given:

- a piston-cylinder device containing H_2O
- $m = 2 \text{ kg}$
- $T_1 = 300^\circ\text{C}$, saturated liquid
- $1 \rightarrow 2$ constant volume.
- $2 \rightarrow 3$ constant pressure at 2 bar.
- $3 \rightarrow 1$ $Pv^{1.1} = \text{constant}$.

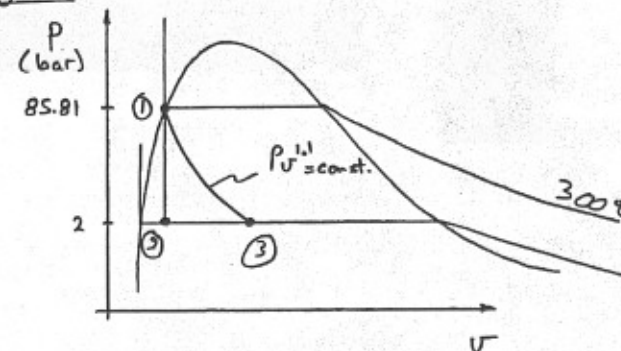
Find:

- Q + W for each process.
- Is it a heat engine or a refrigerator.

Assumptions:

- quasi-equilibrium processes.
- closed system.
- kinetic + potential energy negligible.

Analysis:



Process $1 \rightarrow 2$

Constant volume $\therefore W_2 = 0$

- first law for closed system

$${}_1Q_2 - {}_1W_2 = U_2 - U_1$$

$${}_1Q_2 = m(u_2 - u_1)$$

- need to fix state ① and ②

① saturated liquid at 300°C

$$\begin{aligned} \bullet \bullet \quad u_1 &= 1332.0 \text{ kJ/kg} \\ v_1 &= 0.0014036 \text{ m}^3/\text{kg} \\ p_1 &= 85.81 \text{ bar} \end{aligned} \quad \text{(A-2)}$$

② $p_2 = 2 \text{ bar}$

$$u_2 = u_1 = 0.0014036 \text{ m}^3/\text{kg}$$

$$\bullet \bullet \quad x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.0014036 - 0.0010605}{0.8857 - 0.0010605}$$

$$x_2 = 387.8 \times 10^{-6}$$

Note: extremely low quality because the saturated liquid line is almost vertical.

$$u_2 = u_f + x_2(u_g - u_f) = 504.49 + 387.8 \times 10^{-6} (2529.5 - 504.49)$$

$$u_2 = 505.28 \text{ kJ/kg}$$

$${}_1Q_2 = 2 \text{ [kg]} (505.28 - 1332.0) \left[\frac{\text{kJ}}{\text{kg}} \right]$$

$$\boxed{{}_1Q_2 = -1653.4 \text{ kJ}}$$

Process ② \rightarrow ③

$${}_2W_3 = \int_2^3 p dV = p_2 (V_3 - V_2) \quad \text{constant } p$$

$$= p_2 m (v_3 - v_2)$$

- need v_3 to evaluate ${}_2W_3$

$$p_1 v_1^{1.1} = p_3 v_3^{1.1}$$

$$v_3 = v_1 \left(\frac{p_1}{p_3} \right)^{1/1.1} = 0.0014036 \left(\frac{85.81}{2} \right)^{1/1.1}$$

$$v_3 = 0.04379 \text{ m}^3/\text{kg}$$

$${}_2W_3 = 200 \text{ [kg]} \cdot 2 \left[\frac{\text{kJ}}{\text{kg}} \right] (0.04379 - 0.0014036) \left[\frac{\text{m}^3}{\text{kg}} \right]$$

$$\boxed{{}_2W_3 = 16.55 \text{ kJ}}$$

$${}_2Q_3 = u_3 - u_2 + {}_2W_3 = m(u_3 - u_2) + {}_2W_3$$

- need u_3

$$x_3 = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.04379 - 0.0010605}{0.8857 - 0.0010605} = 0.04717$$

$$u_3 = u_f + x_3(u_g - u_f) = 504.49 + 0.04717(2529.5 - 504.49) = 600.0 \frac{\text{kJ}}{\text{kg}}$$

$${}_2Q_3 = 2 \text{ [kg]} (600.0 - 505.28) \left[\frac{\text{kJ}}{\text{kg}} \right] + 16.55 \text{ [kJ]}$$

$$\boxed{{}_2Q_3 = 206.0 \text{ kJ}}$$

Process ③ \rightarrow ①

$${}_3W_1 = \int_3^1 p dV = \frac{p_1 V_1 - p_3 V_3}{1-n} \quad \text{for a polytropic process.}$$

$${}_3W_1 = \frac{m}{1-n} (P_1 v_1 - P_3 v_3)$$

$$= \frac{2 \text{ [kg]}}{1-1.1} (8581(0.0014036) - 200(0.04279)) \left[\text{kPa} \frac{\text{m}^3}{\text{kg}} \right]$$

$$\boxed{{}_3W_1 = -69.73 \text{ kJ}}$$

$${}_3Q_1 = U_1 - U_3 + {}_3W_1$$

$$= m(u_1 - u_3) + {}_3W_1$$

$$= 2 \text{ [kg]} (1332.0 - 600.0) \left[\frac{\text{kJ}}{\text{kg}} \right] - 69.73 \text{ [kJ]}$$

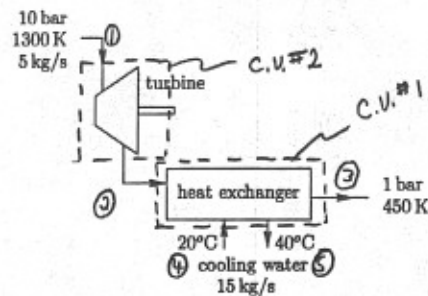
$$\boxed{{}_3Q_1 = 1394 \text{ kJ}}$$

Summary:

	Q (kJ)	W (kJ)
1 → 2	-1653	0
2 → 3	206.0	16.55
3 → 1	1394	-69.73
net	-53	-53

Since $W_{\text{net}} < 0$ the cycle is a "refrigerator."

Given:



$$\text{Air: } R = 286.9 \frac{\text{J}}{\text{kg K}}$$

$$\text{Water: } c_p = 4.18 \frac{\text{kJ}}{\text{kg K}}$$

Find: Power output of the turbine (MW).

Assumptions:

- steady state.
- negligible kinetic + potential energy everywhere.
- no heat transfer in turbine.
- no heat transfer from heat exchanger to surroundings.
- air is an ideal gas.
- cooling water has a constant c_p .

Analysis:

Consider the Heat Exchanger (C.V. #1)

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{c.v.}} - \dot{W}_{\text{c.v.}} + \sum_{\text{inlets}} \dot{m}_i \left(h + \frac{V^2}{2} + gz \right)_i - \sum_{\text{exits}} \dot{m}_e \left(h + \frac{V^2}{2} + gz \right)_e$$

$$0 = \dot{m}_2 h_2 + \dot{m}_4 h_4 - \dot{m}_3 h_3 - \dot{m}_5 h_5$$

$$\dot{m}_2 = \dot{m}_3 = \dot{m}_{\text{air}}$$

$$\dot{m}_4 = \dot{m}_5 = \dot{m}_{\text{H}_2\text{O}}$$

$$0 = \dot{m}_{\text{air}} (h_3 - h_5) + \dot{m}_{\text{H}_2\text{O}} (h_4 - h_5)$$

$$\text{at } T_3 = 450 \text{ K}, \quad h_3 = 451.80 \frac{\text{kJ}}{\text{kg}} \quad (\text{A-22})$$

$$\text{also, } h_4 - h_5 = c_p (T_4 - T_5)$$

$$0 = 5 \left[\frac{\text{kJ}}{\text{s}} \right] (h_2 - 451.80 \frac{\text{kJ}}{\text{kg}}) + 15 \left[\frac{\text{kJ}}{\text{s}} \right] 4.18 \frac{\text{kJ}}{\text{kg K}} (20 - 40) [\text{K}]$$

$$h_2 = 702.6 \frac{\text{kJ}}{\text{kg}}$$

Turbine (C.U.#2)

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_{\text{inlets}} \dot{m}_i \left(h_i \frac{V_i}{2} + g z_i \right) - \sum_{\text{exits}} \dot{m}_e \left(h_e \frac{V_e}{2} + g z_e \right)$$

$$\dot{W}_t = \dot{m}_{\text{air}} (h_1 - h_2)$$

$$\text{at } T_1 = 1300 \text{ K}, \quad h_1 = 1395.97 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

$$\dot{W}_t = 5 \left[\frac{\text{kJ}}{\text{s}} \right] (1395.97 - 702.6) \left[\frac{\text{kJ}}{\text{kg}} \right]$$

$$\dot{W}_t = 3.47 \text{ MW}$$

Alternate Solution

- consider a C.U. around both components

$$\frac{dE_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \sum_{\text{inlets}} \dot{m}_i \left(h_i \frac{V_i}{2} + g z_i \right) - \sum_{\text{exits}} \dot{m}_e \left(h_e \frac{V_e}{2} + g z_e \right)$$

$$\dot{W}_t = \dot{m}_2 (h_1 - h_2) + \dot{m}_{\text{H}_2\text{O}} (h_4 - h_5)$$

$$= 5 \left[\frac{\text{kJ}}{\text{s}} \right] (1395.97 - 702.6) \left[\frac{\text{kJ}}{\text{kg}} \right] + 15 \left[\frac{\text{kJ}}{\text{s}} \right] 4.18 \left[\frac{\text{kJ}}{\text{kg K}} \right] (-20) [\text{K}]$$

$$\dot{W}_t = 3.47 \text{ MW}$$