M E 227.3 Thermodynamics I
Department of Mechanical Engineering
Midterm Examination
October 27, 2004

Time: 90 minutes
Calculators Allowed
Formula Sheet Supplied

Total Marks: 50
Closed Book
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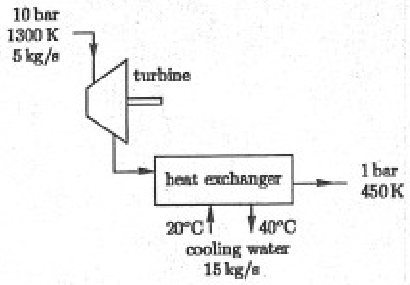
(10) 1. Define the following terms.

(a) Enthalpy
(b) Total Energy
(c) Ideal Gas
(d) Saturated Vapour
(e) Isobaric Process

(20) 2. A piston-cylinder device contains 2 kg of water which is initially a saturated liquid at 300°C. It undergoes three processes to form a cycle. The first process is constant volume, the second is constant pressure at 2 bar and the third is a polytropic process where $Pv^{1.1}$ = constant. Calculate heat transfer and work (both in kJ) for each process. Is the device a heat engine or a refrigerator?

(20) 3. Air ($R = 286.9\,\mathrm{J/(kg \cdot K)}$) at 1300 K and 10 bar enters a turbine with a mass flowrate of 5 kg/s. After exiting the turbine, the air enters a heat exchanger where it is cooled to 450 K before discharging to the atmosphere at 1 bar. The heat exchanger warms liquid water flowing at 15 kg/s from 20°C to 40°C. Calculate the power output of the turbine (MW). For the cooling water, you may assume that $c_P = 4.18\,\mathrm{kJ/(kg \cdot K)}$ is a constant.



Given:

- a piston-cylinder device containing H_2O
- $m = 2$ kg
- $T_1 = 300°C$, saturated liquid
- $1 \rightarrow 2$ constant volume.
  $2 \rightarrow 3$ constant pressure at 2 bar.
  $3 \rightarrow 1$ $Pv^{1.1}$ = constant.

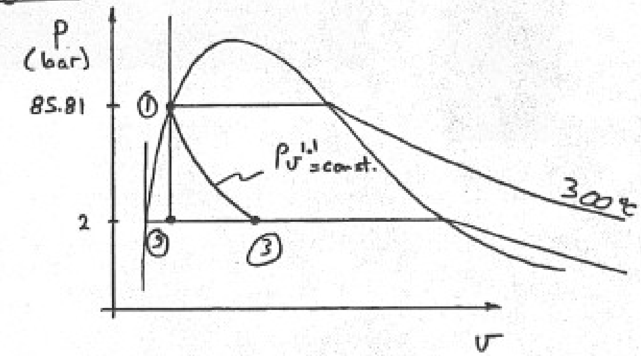
Find: (a) Q + W for each process.

(b) Is it a heat engine or a refrigerator.

Assumptions:

- quasi-equilibrium processes.
- closed system.
- Kinetic + potential energy negligible.

Analysis:



Process ①→②

Constant volume $\therefore$ $\boxed{{}_1W_2 = 0}$

- first law for closed system

$${}_1Q_2 - \cancel{{}_1W_2}^{\,0} = U_2 - U_1$$

$${}_1Q_2 = m(u_2 - u_1)$$

- need to fix state 1 and 2

1 saturated liquid at 300°C

$$\therefore \left.\begin{aligned} u_1 &= 1332.0 \text{ kJ/kg} \\ v_1 &= 0.0014036 \text{ m}^3/\text{kg} \\ P_1 &= 85.81 \text{ bar} \end{aligned}\right\} (A\text{-}2)$$

2 $P_2 = 2$ bar

$v_2 = v_1 = 0.0014036 \text{ m}^3/\text{kg}$

$$\therefore x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.0014036 - 0.0010605}{0.8857 - 0.0010605}$$

$$x_2 = 387.8 \times 10^{-6}$$

<u>Note:</u> extremely low quality because the saturated liquid line is almost vertical.

$$u_2 = u_f + x_2(u_g - u_f) = 504.49 + 387.8\times10^{-6}(2529.5 - 504.49)$$

$$u_2 = 505.28 \text{ kJ/kg}$$

$${}_1Q_2 = 2[\text{kg}]\,(505.28 - 1332.0)\left[\frac{\text{kJ}}{\text{kg}}\right]$$

$$\boxed{{}_1Q_2 = -1653.4 \text{ kJ}}$$

Process 2 → 3

$${}_2W_3 = \int_2^3 P\,dV = P_2(V_3 - V_2) \quad \text{constant } P$$

$$= P_2\, m\,(v_3 - v_2)$$

- need v_3 to evaluate ${}_2W_3$

$$P_1 v_1^{1.1} = P_3 v_3^{1.1}$$

$$v_3 = v_1\left(\frac{P_1}{P_3}\right)^{\frac{1}{1.1}} = 0.0014036\left(\frac{85.81}{2}\right)^{\frac{1}{1.1}}$$

$$v_3 = 0.04279 \text{ m}^3/\text{kg}$$

$${}_2W_3 = 200[\text{kPa}]\; 2[\text{kg}]\,(0.04279 - 0.0014036)\left[\frac{\text{m}^3}{\text{kg}}\right]$$

$$\boxed{{}_2W_3 = 16.55 \text{ kJ}}$$

$${}_2Q_3 = U_3 - U_2 + {}_2W_3 = m(u_3 - u_2) + {}_2W_3$$

- need u_3

$$x_3 = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.04279 - 0.001065}{0.8857 - 0.001065} = 0.04717$$

$$u_3 = u_f + x_3(u_g - u_f) = 504.49 + 0.04717(2529.5 - 504.49) = 600.0 \text{ kJ/kg}$$

$${}_2Q_3 = 2[\text{kg}]\,(600.0 - 505.28)\left[\frac{\text{kJ}}{\text{kg}}\right] + 16.55\,[\text{kJ}]$$

$$\boxed{{}_2Q_3 = 206.0 \text{ kJ}}$$

Process 3 → 1.

$${}_3W_1 = \int_3^1 P\,dV = \frac{P_1V_1 - P_3V_3}{1-n} \quad \text{for a polytropic process,}$$

$$_3W_1 = \frac{m\left(P_1 v_1 - P_3 v_3\right)}{1-n}$$

$$= \frac{2\,[kg]}{1-1.1}\left(8581(0.0014036) - 200(0.04279)\right)\left[kPa\,\frac{m^3}{kg}\right]$$

$$\boxed{_3W_1 = -69.73\ kJ}$$

$$_3Q_1 = U_1 - U_3 + {}_3W_1$$

$$= m(u_1 - u_3) + {}_3W_1$$

$$= 2\,[kg]\left(1332.0 - 600.0\right)\left[\frac{kJ}{kg}\right] - 69.73\,[kJ]$$

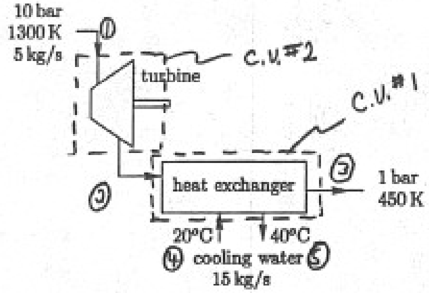
$$\boxed{_3Q_1 = 1394\ kJ}$$

Summary:

	Q (kJ)	W (kJ)
1→2	-1653	0
2→3	206.0	16.55
3→1	1394	-69.73
net	-53	-53

Since $W_{net} < 0$ the cycle is a "refrigerator."

Given:



Air: $R = 286.9\ \frac{J}{kg\,K}$

Water: $C_p = 4.18\ \frac{kJ}{kg\,K}$

Find: Power output of the turbine (MW).

Assumptions:

- steady state.
- negligible kinetic + potential energy everywhere.
- no heat transfer in turbine.
- no heat transfer from heat exchanger to surroundings.
- air is an ideal gas.
- Cooling water has a constant Cp.

Analysis:

Consider the Heat Exchanger (C.V. #1)

$$\cancel{\frac{dE_{c.v.}}{dt}}^{0} = \cancel{\dot{Q}_{c.v.}}^{0} - \cancel{\dot{W}_{c.v.}}^{0} + \sum_{inlets} \dot{m}_i\left(h + \cancel{\frac{V^2}{2}}^{0} + \cancel{gz}^{0}\right)_i - \sum_{exits} \dot{m}_e\left(h + \cancel{\frac{V^2}{2}}^{0} + \cancel{gz}^{0}\right)_e$$

$$0 = \dot{m}_2 h_2 + \dot{m}_4 h_4 - \dot{m}_3 h_3 - \dot{m}_5 h_5$$

$\dot{m}_2 = \dot{m}_3 = \dot{m}_{air}$

$\dot{m}_4 = \dot{m}_5 = \dot{m}_{H_2O}$

$$0 = \dot{m}_{air}(h_2 - h_3) + \dot{m}_{H_2O}(h_4 - h_5)$$

at $T_3 = 450\,K$, $h_3 = 451.80 \frac{kJ}{kg}$ (A-22)

also, $h_4 - h_5 = c_p(T_4 - T_5)$

$$0 = 5\left[\frac{kg}{s}\right]\left(h_2 - 451.80\right)\left[\frac{kJ}{kg}\right] + 15\left[\frac{kg}{s}\right]4.18\left[\frac{kJ}{kg\,K}\right](20-40)[K]$$

$$h_2 = 702.6 \frac{kJ}{kg}$$

Turbine (C.V. #2)

$$\frac{dE_{C.V.}}{dt} = \dot{Q}_{C.V.} - \dot{W}_{C.V.} + \sum_{inlets} \dot{m}_i\left(h + \frac{V^2}{2} + gz\right)_i - \sum_{exits} \dot{m}_e\left(h + \frac{V^2}{2} + gz\right)_e$$

$$\dot{W}_t = \dot{m}_{air}(h_1 - h_2)$$

at $T_1 = 1300\,K$, $h_1 = 1395.97 \left[\frac{kJ}{kg}\right]$

$$\dot{W}_t = 5\left[\frac{kg}{s}\right]\left(1395.97 - 702.6\right)\left[\frac{kJ}{kg}\right]$$

$$\boxed{\dot{W}_t = 3.47\ MW}$$

Alternate Solution

- consider a C.V. around both components

$$\frac{dE_{C.V.}}{dt} = \dot{Q}_{C.V.} - \dot{W}_{C.V.} + \sum_{inlets} \dot{m}_i\left(h + \frac{V^2}{2} + gz\right)_i - \sum_{exits} \dot{m}_e\left(h + \frac{V^2}{2} + gz\right)_e$$

$$\dot{W}_t = \dot{m}_a(h_1 - h_3) + \dot{m}_{H_2O}(h_4 - h_5)$$

$$= 5\left[\frac{kg}{s}\right]\left(1395.97 - 451.80\right)\left[\frac{kJ}{kg}\right] + 15\left[\frac{kg}{s}\right]4.18\left[\frac{kJ}{kg\,K}\right](-20)[K]$$

$$\boxed{\dot{W}_t = 3.47\ MW}$$