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Name: [redacted]

ID #: [redacted]

University of Saskatchewan
Department of Mechanical Engineering
ME 251 Engineering Analysis I

February 22, 2005 | Mid Term Exam | 90 minutes

Open Book Exam
Answer All Questions and Show All Work
Answer All Descriptive Questions Clearly and Concisely

9

5 Questions - 50 Points Total

1. a) Briefly describe two ways in which standards are important when conducting a failure analysis, such as the investigation into the collapse of the World Trade Center towers on September 11, 2001.

5 (5)

could be more concise

Standards can be used to see if the engineers of WTC did all they could. They designed the WTC to withstand fire and aircraft collision but not at the same time, ie. the fire protectant on the steel was blown off by the airplane, or the shattered gyprock. I believe the engineers did all they could within reason and without building a bunker. They can't think of every possible thing happening but the good thing with standards is that they can now be updated to require taking these types of attacks into consideration.

block stairs

b) Briefly identify two reasons why many practical engineering problems must be solved using numerical analysis.

4 (5)

Normally, the cause of using numerical analysis is because the equations would be too difficult to solve analytically.

Some areas include multidisciplinary problems (thermal/structural), when the material property is not constant, and finite element analysis.

Also, many practical engineering problems aren't exact anyway.

and

example of num'l anal' vs. reason for doing num'l anal'

clarify

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2. After completing a derivation, an engineer is left with the following system of linear equations:

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(10) $$[A]\{x\} = \{R\}$$

Let:

$$[A] = \begin{bmatrix} 5 & 2 \\ 8 & 4 \end{bmatrix}, \{x\} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \text{ and } \{R\} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$$

Solve this series of linear equations for x_1 and x_2 **using Doolittle's method.**

L U decomp

$$[L][U] = \begin{bmatrix} 5 & 2 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

$5 = u_{11}$

$2 = u_{12}$

$8 = L_{21} u_{11} \rightarrow 8 = 5 L_{21}$

$L_{21} = \frac{8}{5}$

$4 = L_{21} u_{12} + u_{22}$

$4 = \frac{8}{5}(2) + u_{22}$

$u_{22} = 4 - \frac{16}{5}$

$u_{22} = \frac{4}{5}$

$[L][y] = [b]$

$$\begin{bmatrix} 1 & 0 \\ 8/5 & 1 \end{bmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} 9 \\ 16 \end{bmatrix}$$

$y_1 = 9$

$\frac{8}{5} y_1 + y_2 = 16$

$\frac{8}{5}(9) + y_2 = 16$

$y_2 = 1.6$

$$\{y\} = \begin{bmatrix} 9 \\ 1.6 \end{bmatrix}$$

$[U][x] = [y]$

$$\begin{bmatrix} 5 & 2 \\ 0 & 4/5 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 9 \\ 1.6 \end{bmatrix}$$

$\frac{4}{5} x_2 = 1.6$

$x_2 = 2$

$5x_1 + 2x_2 = 9$

$5x_1 + 2(2) = 9$

$x_1 = 1$

$$\boxed{X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

check

$5 + 4 = 9$

$8 + 8 = 16$

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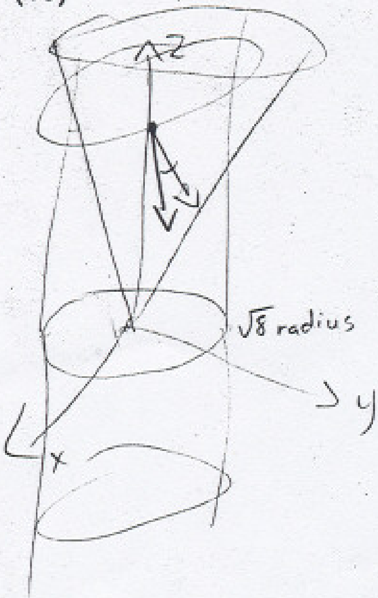
3. Determine the angle between the normal vectors to the following two surfaces at the point $(2,2,\sqrt{8})$: 45°

(10)



$z = \sqrt{x^2 + y^2}$

$z^2 = x^2 + y^2$ $\quad 0 = x^2 + y^2 - z^2 = f(x,y,z)$

$x^2 + y^2 = 8$ $\quad 0 = x^2 + y^2 - 8 = g(x,y,z)$

gradient gives normal vectors

cone and cylinder

∇

$\nabla f(x,y,z) = (2x, 2y, -2z)$

$\nabla f(2,2,\sqrt{8}) = (4, 4, -2\sqrt{8}) = \vec{a}$ $\qquad |\vec{a}| = 8$

$\nabla g(x,y,z) = (2x, 2y, 0)$

$\nabla g(2,2,\sqrt{8}) = (4, 4, 0) = \vec{b}$ $\qquad |\vec{b}| = 2\sqrt{8}$

dot product

$\vec{a} \cdot \vec{b} = ab \cos\theta$

$(4, 4, -2\sqrt{8}) \cdot (4, 4, 0) = 8(2\sqrt{8}) \cos\theta$

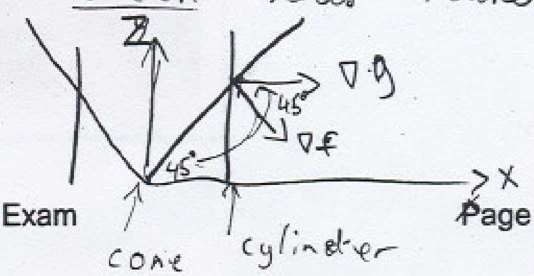
$16 + 16 + 0 = 16\sqrt{8} \cos\theta$

$32 = 16\sqrt{8} \cos\theta$

$\frac{2}{\sqrt{8}} = \cos\theta$

$\boxed{\theta = 45^\circ}$

check this makes sense



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4. The velocity of a fluid flow is given by

$$\bar{v} = (xz - y)\,\vec{i} + yz\,\vec{j} + 2xy\,\vec{k}$$

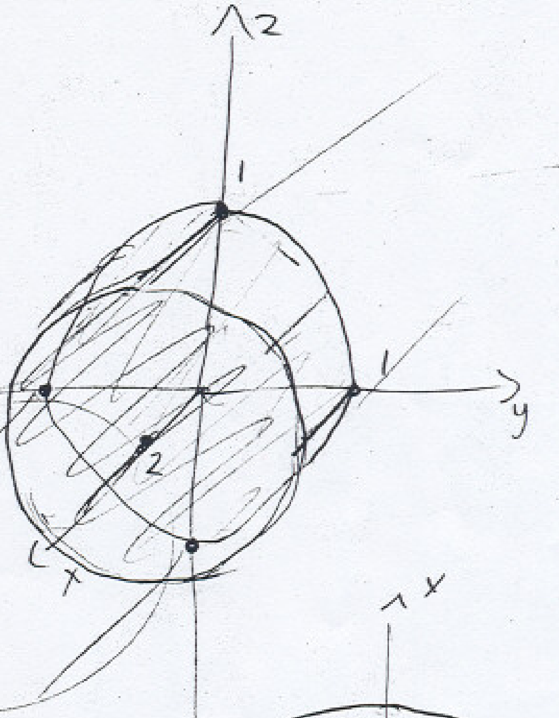
(10)

Calculate the flux of the velocity through the surface S, where S is the surface of the region Q that lies between the cylinder $y^2 + z^2 = 1$ and the planes x = 0 and x = 2.

$z = \pm\sqrt{1-y^2}$

Flux so Divergence theorem

$$\iint_S (\vec{v} \cdot \hat{n})\, dA = \iiint_Q \nabla \cdot \vec{v}\, dV$$



cylindrical coordinates

$$\nabla \cdot \vec{v} = z + z + 0$$

$$\nabla \cdot \vec{v} = 2z$$

$z = r\sin\theta$ - see sol'ns

$$\boxed{\int_{x=0}^{2}\int_{\theta=0}^{2\pi}\int_{r=0}^{1} 2z \quad r\,dr\,d\theta\,dx}$$

$$\int_{x=0}^{2}\int_{\theta=0}^{2\pi} \left[z r^2\right]_0^1 d\theta\, dx$$

$$\int_{x=0}^{2}\int_{\theta=0}^{2\pi} z \; d\theta\, dx$$

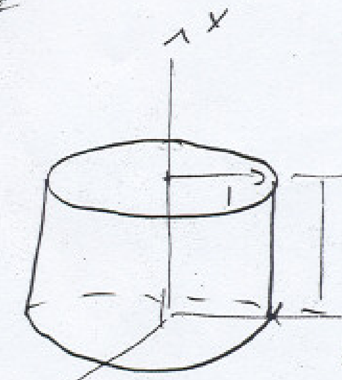
$$\int_{x=0}^{2} \left[z\theta\right]_0^{2\pi} dx$$

$$\int_{x=0}^{2} \left[2\pi z\right] dx$$

$$= \left[2\pi z x\right]_0^2 = 4\pi z$$

omit

I don't know how to put z in terms of x



$$\int_{x=0}^{2}\int_{y=-1}^{1}\int_{z=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 2z\; dz\,dy\,dx$$

$$= \quad ''\; ''$$