Student Number: ____________________ Name: ______________________________

Midterm Exam

M E 251.3 – ENGINEERING ANALYSIS I
Department of Mechanical Engineering
University of Saskatchewan

7:00 – 9:00pm, Wednesday, February 21, 2007

Instructor: Prof. FangXiang Wu

PLEASE READ CAREFULLY:

This exam has 5 problems in two pages. The exam is CLOSED book. Some useful formulas and table are on the first page. A scientific calculator is permitted just for problems 4 and 5 and results using calculators for other problems are unacceptable. Please attempt all 5 problems and ensure that your answers are clear and legible. You must return the question sheets together with your examination booklet(s).

- Newton's forward divided difference formula

$$f(x) \approx p_n(x) = f_0 + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$$
$$+ (x - x_0)(x - x_1)\ldots(x - x_{n-1}) f[x_0, x_1, \ldots, x_n]$$

- Newton's forward difference formula

$$f(x) \approx P_n(x) = \sum_{s=0}^{n} \binom{r}{s} \Delta^s f_0 = f_0 + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + \ldots + \frac{r(r-1)\ldots(r-n+1)}{n!}\Delta^n f_0$$

- Gauss Integration table for n=3 nodes:

n	**Nodes t_j**	**Weights A_j**
3	**− 0.7746**	**5/9**
	0	**8/9**
	0.7746	**5/9**

Problem 1 (20 points) Answer the following questions.

a) What are the elementary row operations?

b) What is the Gauss-Seidel iteration formula for numerically solving the linear system $[A]\vec{x} = \vec{b}$? Please clearly define the notations in your answer.

c) Using the notations in b), what are the convergence condition and the stability condition of the Gauss-Seidel iteration method?

d) What are the convergence conditions of the brute-force formula for numerically solving the nonlinear equation $x = g(x)$?

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Problem 2 **(20 points)** After completing a derivation, an engineer is left with the following system of linear equations:

$$[A]\vec{x}=\vec{b}$$

where:

$$[A]=\begin{bmatrix}3 & 4 & 3\\ 1 & 2 & 0\\ 4 & 5 & 3\end{bmatrix},\ \vec{x}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix},\ \text{and}\ \vec{b}=\begin{bmatrix}20\\ 10\\ 30\end{bmatrix}$$

a) Calculate the determinant of [A] by expanding along a row or a column;
b) Solve this system of linear equations using Gauss elimination method.

Problem 3 **(20 points)** A Markov process has the state transition matrix

$$[T]=\begin{bmatrix}0.9 & 0.5\\ 0.1 & 0.5\end{bmatrix}$$

Find the limit state $\vec{x}_\infty=\lim_{n\to\infty}[T]^n\vec{x}_0$ for an arbitrary initial state $\vec{x}_0=\begin{bmatrix}x_{10} & x_{20}\end{bmatrix}^T$, where $x_{10}, x_{20}\geq 0$, and $x_{10}+x_{20}=1$.

Problem 4 **(25 points)** Given the following table of data:

x	0.4	0.6	0.8	1.0
f(x)	0.428	0.604	0.742	0.843

Using an interpolation method and all given data above

a) Find the value of f(0.7);

b) Find the approximate root of equation f(x) = 0.7;

(Show only 4 decimal places in your answer.)

Problem 5 **(15 points)** Numerically calculate the integral

$$J=\int_0^2\frac{1}{x^2-2x+2}dx$$

Using the following numerical method:

a) Simpson's formula with 4 subintervals (i.e., $h=0.5$)
b) Gauss integration formula with 3 nodes.

(Show only 4 decimal places in your answer.)