

**UNIVERSITY OF SASKATCHEWAN  
ME 313.3 – MECHANICS OF MATERIALS I  
FINAL EXAM – DECEMBER 6, 2007**

**Professor A. Dolovich**

**A CLOSED BOOK EXAMINATION  
TIME: 3 HOURS**

LAST NAME (printed): \_\_\_\_\_

FIRST NAME (printed): \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

EXAMINATION ROOM: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

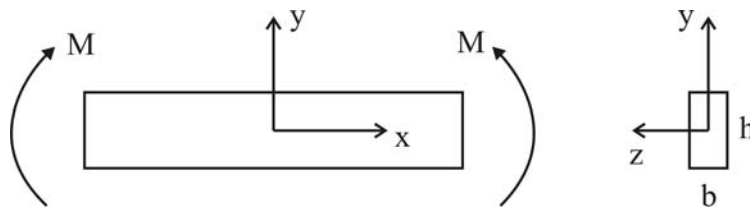
**INSTRUCTIONS**

For Marker's  
Use Only

- |  |   |
|--|---|
| 1) The examination consists of 14 questions.<br>Answer all FOURTEEN questions.<br>The exam is out of a total of 100 marks.<br>The marks for each question is given in brackets.<br>PRINT YOUR NAME AT THE TOP OF EACH PAGE.  | 1. _____<br>2. _____<br>3. _____<br>4. _____  |
| 2) This is a closed book exam.<br>Calculators are permitted.<br>A list of formulas will be provided separately.  | 5. _____<br>6. _____  |
| 3) SHOW YOUR WORK AND ANSWERS CLEARLY.<br>Give all final numerical answers to 3 significant figures.<br>Give your solutions in the space below the question.<br>Neatly place a box around your final answers.<br>The back of each page may be used as a continuation<br>sheet if required. | 7. _____<br>8. _____<br>9. _____<br>10. _____<br>11. _____<br>12. _____<br>13. _____<br>14. _____ |

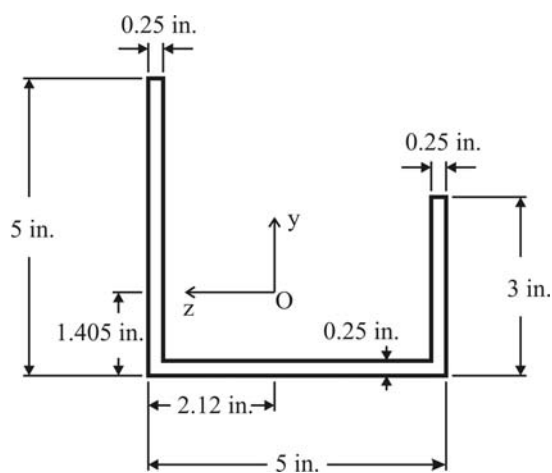
Total:

- (8) 1. A bending moment,  $M$ , is applied to a beam of constant rectangular cross section, with width  $b$  and depth  $h$ . The beam is made of a nonlinear material for which the stress-strain relationship is  $\sigma = k\varepsilon^3$ . The neutral axis (i.e., the  $z$  axis) is located a distance  $h/2$  from the bottom of the cross section.



Obtain an expression for the normal stress  $\sigma_x$  in the beam. Write your answer in terms of  $M$ ,  $y$ ,  $b$ , and  $h$ .

- (4) 2. For the cross section given below, determine the area product of inertia,  $I_{yz}$ , with respect to the y-z coordinate system defined in the sketch. The centroid O is located 2.12 inches to the right and 1.405 inches above the bottom left corner of the cross section, as shown.

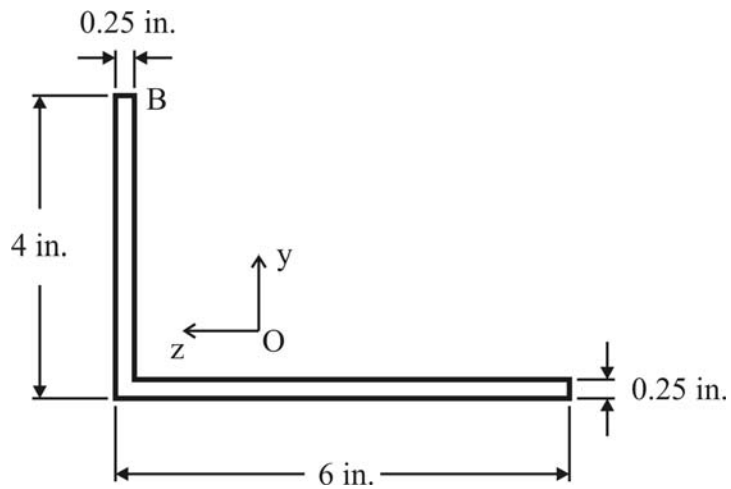


- (12) 3. A cross section with centroid O carries a bending moment  $M_z = 15000$  in·lb (with the moment vector in the positive z direction). The area moments of inertia  $I_{yy}$  and  $I_{zz}$ , as well as the area product of inertia  $I_{yz}$ , are given below, together with a sketch of the cross section. Determine the normal stress  $\sigma_x$  at point B which is located at coordinates  $(y,z) = (3.58$  in.,  $1.644$  in.).

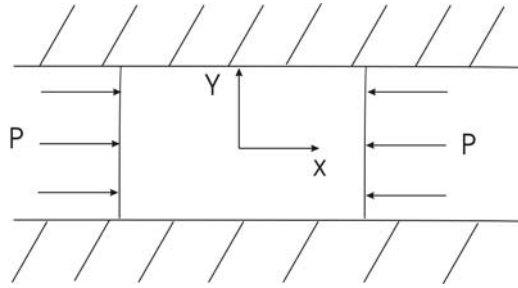
$$I_{yy} = 9.27 \text{ in}^4$$

$$I_{zz} = 3.41 \text{ in}^4$$

$$I_{yz} = 3.32 \text{ in}^4$$



4. A block is subjected to equal and opposite pressures,  $P$ , along the  $x$ -direction, and is constrained from expanding in the  $y$  direction by two smooth walls. Both the stress and strain fields are uniform and the block is made of a linear elastic, homogenous, isotropic material with Young's modulus,  $E$ , and Poisson's ratio,  $\nu$ .



There is no temperature change and the body is free to expand in the  $z$ -direction.

- (4) (a) Find  $\epsilon_x$  and  $\epsilon_z$  in terms of  $P$ ,  $E$  and  $\nu$ .
- (4) (b) If, in addition to smooth walls constraining the block in the  $y$ -direction, there were also smooth walls constraining the block in the  $z$ -direction, and the block was subjected to a temperature change  $\Delta T$ , determine the magnitude of  $P$  required so that  $\epsilon_x$  would be zero. The coefficient of thermal expansion for the block is  $\alpha$ . Write your answer in terms of  $E$ ,  $\nu$ ,  $\alpha$ , and  $\Delta T$ .

5. A thin plate lies in the x-y plane and is subjected to applied loads which produce the displacement field

$$u = (c_1y + c_2xy + c_3x^2) \times 10^{-4}$$

$$v = (c_4x + c_5yx + c_6y^2) \times 10^{-4}$$

where  $u$ ,  $v$ ,  $x$ , and  $y$  are in meters,  $c_1 = 0.75$ ,  $c_2 = 2.4 \text{ m}^{-1}$ ,  $c_4 = 1.3$ ,  $c_5 = 0.6 \text{ m}^{-1}$ , and  $c_3$  and  $c_6$  are unknown. The plate is in a state of plane stress, with  $\sigma_{zz} = 0 = \tau_{zx} = 0 = \tau_{zy}$  and negligible body forces. Also, the plate is made of a linearly elastic material with a Young's modulus,  $E$ , and a Poisson's ratio,  $\nu = 0$ . There is no temperature change  $\Delta T$ .

- (8) (a) Find the coefficients  $c_3$  and  $c_6$  that satisfy equilibrium.

- (2) (b) Is compatibility satisfied? Why or why not?

6. A plate in the x-y plane is linear elastic with  $E = 70$  GPa,  $\nu = 0.3$  and a displacement field

$$u = (c_1x^2 + c_2y + c_3xy^2 + c_4y^3) \times 10^{-5}$$

$$v = (c_5y + c_6x^3y^2 + c_7xy + c_8y^2) \times 10^{-5}$$

where  $c_1 = 5.1 \text{ m}^{-1}$ ,  $c_2 = 7.3$ ,  $c_3 = 4.2 \text{ m}^{-2}$ ,  $c_4 = 8.5 \text{ m}^{-2}$ ,  $c_5 = 2.7$ ,  $c_6 = 3.9 \text{ m}^{-4}$ ,  $c_7 = 6.8 \text{ m}^{-1}$  and  $c_8 = 5.4 \text{ m}^{-1}$ .

- (4) (a) Find the rigid body rotation at  $x = 1$ ,  $y = 2$ .

- (4) (b) Find the total change in angle of a line element at  $x = 2$ ,  $y = 3$  where the line element is oriented  $20^\circ$  CCW (counter-clockwise) from the x-direction.

- (2) 7. The stress state at a point with respect to an x, y, z coordinate system is given by

$$[\sigma]_{x,y,z} = \begin{bmatrix} 3 & 6 & 0 \\ 6 & 9 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ MPa.}$$

If you wanted to determine the normal stress on a plane whose outward normal is oriented at angles  $40^\circ$ ,  $60^\circ$ , and  $66.2^\circ$  with respect to the x, y and z axes, respectively, which method would you use? (Please circle one).

- (a) Eigenvalue method
- (b) Mohr's circle
- (c) Cosine transformation law
- (4) 8. Given a uniform state of plane stress where  $\Theta_{xy} = 52.6\mu$  and  $\frac{\partial u}{\partial y} = 13.9\mu$ , find the change in angle of a line element in the x-direction.

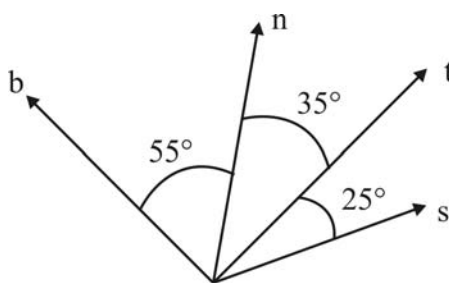
- (6) 9. A point in a loaded body is in a state of plane strain. At this point we have

$$\epsilon_s = 68.2 \mu$$

$$\epsilon_t = 32.7 \mu$$

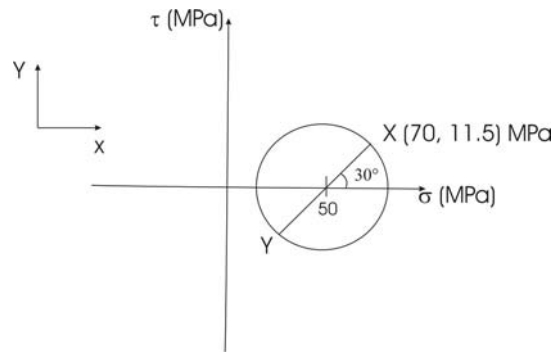
$$\epsilon_n = -34.14 \mu$$

$$\epsilon_b = 59.1 \mu$$



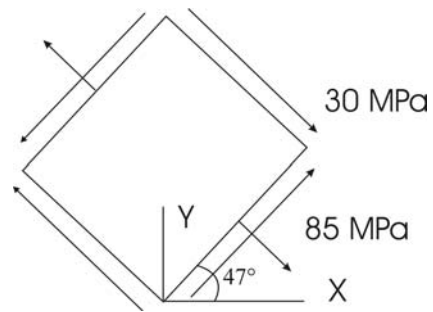
Find  $\gamma_{sn}$ .

10. A body is in a state of plane stress. The following Mohr's circle represents the stress experienced at a particular point.



- (4) (a) Show the stresses corresponding to the x-y coordinate system on a properly oriented element.
- (2) (b) If the body is not in a uniform state of plane stress, what conclusions (if any) could you make about the stresses at other points in the body?

- (4) 11. A body is loaded in a state of plane stress as shown.



Determine the stresses on a plane perpendicular to the x-axis using the cosine transformation law.

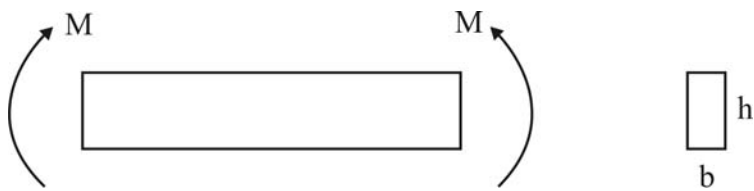
12. The stress state at a point with respect to an x, y coordinate system is given by

$$[\sigma]_{x,y} = \begin{bmatrix} 0 & 25 \\ 25 & 0 \end{bmatrix} \text{MPa.}$$

(4) (a) Using Mohr's circle what is  $\tau_{\max}$ ?

(4) (b) Find  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$ .

13. A beam is subjected to a bending moment as shown.



Knowing that for the beam, the Young's modulus in tension,  $E_t$ , is twice the Young's modulus in compression,  $E_c$ ,

- (6) (a) find the location of the neutral axis from the bottom of the beam.
- (6) (b) Find the formula that gives the bending moment,  $M$ , in terms of the curvature of the beam,  $\rho$ , as well as  $E_t$ ,  $b$  and  $h$ .

- (8) 14. In class we derived the formula for  $\epsilon_x = \partial u / \partial x$  which is valid for geometric linearity and therefore deformation where all changes in angle are very small. Keeping the definition of  $\epsilon_x$  as the change in length divided by the original length of a line element originally along the x-direction, derive the formula for  $\epsilon_x$  in terms of displacement derivatives for the geometrically nonlinear case where angular changes are large.

**End of Exam**