UNIVERSITY OF SASKATCHEWAN
ME 313.3 – MECHANICS OF MATERIALS I
FINAL EXAM – DECEMBER 6, 2007

Professor A. Dolovich

A CLOSED BOOK EXAMINATION
TIME: 3 HOURS

LAST NAME (printed): ________________________________

FIRST NAME (printed): ________________________________

STUDENT NUMBER: ________________________________

EXAMINATION ROOM: ________________________________

SIGNATURE: ________________________________

INSTRUCTIONS

1) The examination consists of 14 questions.
   Answer all FOURTEEN questions.
   The exam is out of a total of 100 marks.
   The marks for each question is given in brackets.
   PRINT YOUR NAME AT THE TOP OF EACH PAGE.

2) This is a closed book exam.
   Calculators are permitted.
   A list of formulas will be provided separately.

3) SHOW YOUR WORK AND ANSWERS CLEARLY.
   Give all final numerical answers to 3 significant figures.
   Give your solutions in the space below the question.
   Neatly place a box around your final answers.
   The back of each page may be used as a continuation sheet if required.

For Marker's Use Only

1. __________
2. __________
3. __________
4. __________
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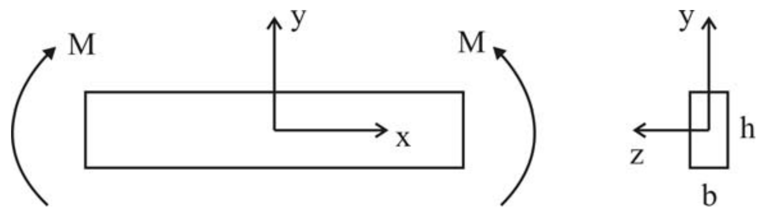
9. __________
10. __________
11. __________
12. __________
13. __________
14. __________

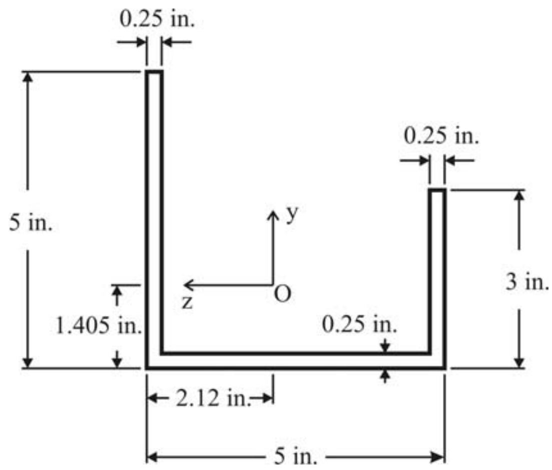
Total:

(8) 1. A bending moment, M, is applied to a beam of constant rectangular cross section, with width b and depth h. The beam is made of a nonlinear material for which the stress-strain relationship is $\sigma = k\varepsilon^3$. The neutral axis (i.e., the z axis) is located a distance h/2 from the bottom of the cross section.



Obtain an expression for the normal stress σ_x in the beam. Write your answer in terms of M, y, b, and h.

(4) 2. For the cross section given below, determine the area product of inertia, I_{yz}, with respect to the y-z coordinate system defined in the sketch. The centroid O is located 2.12 inches to the right and 1.405 inches above the bottom left corner of the cross section, as shown.

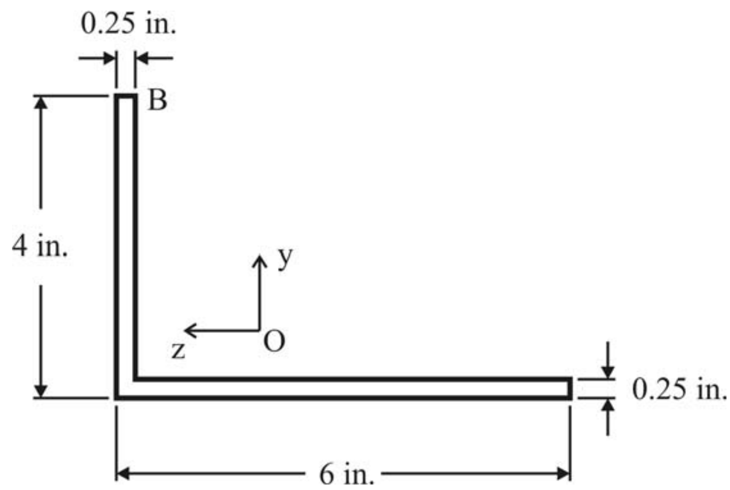


(12) 3. A cross section with centroid O carries a bending moment M_z = 15000 in·lb (with the moment vector in the positive z direction). The area moments of inertia I_{yy} and I_{zz}, as well as the area product of inertia I_{yz}, are given below, together with a sketch of the cross section. Determine the normal stress σ_x at point B which is located at coordinates (y,z) = (3.58 in., 1.644 in.).

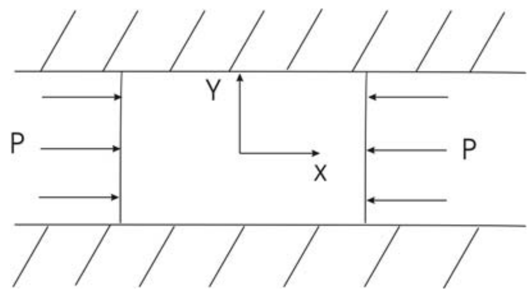
$I_{yy} = 9.27\ \text{in}^4$

$I_{zz} = 3.41\ \text{in}^4$

$I_{yz} = 3.32\ \text{in}^4$



4. A block is subjected to equal and opposite pressures, P, along the x-direction, and is constrained from expanding in the y direction by two smooth walls. Both the stress and strain fields are uniform and the block is made of a linear elastic, homogenous, isotropic material with Young's modulus, E, and Poisson's ratio, ν.



There is no temperature change and the body is free to expand in the z-direction.

(4) (a) Find ε_x and ε_z in terms of P, E and ν.

(4) (b) If, in addition to smooth walls constraining the block in the y-direction, there were also smooth walls constraining the block in the z-direction, and the block was subjected to a temperature change ΔT, determine the magnitude of P required so that ε_x would be zero. The coefficient of thermal expansion for the block is α. Write your answer in terms of E, ν, α, and ΔT.

5. A thin plate lies in the x-y plane and is subjected to applied loads which produce the displacement field

$$u = (c_1 y + c_2 xy + c_3 x^2)\times 10^{-4}$$

$$v = (c_4 x + c_5 yx + c_6 y^2)\times 10^{-4}$$

where u, v, x, and y are in meters, $c_1 = 0.75$, $c_2 = 2.4\ m^{-1}$, $c_4 = 1.3$, $c_5 = 0.6\ m^{-1}$, and c_3 and c_6 are unknown. The plate is in a state of plane stress, with $\sigma_{zz} = 0 = \tau_{zx} = 0 = \tau_{zy}$ and negligible body forces. Also, the plate is made of a linearly elastic material with a Young's modulus, E, and a Poisson's ratio, $\nu = 0$. There is no temperature change ΔT.

(8) (a) Find the coefficients c_3 and c_6 that satisfy equilibrium.

(2) (b) Is compatibility satisfied? Why or why not?

6. A plate in the x-y plane is linear elastic with E = 70 GPa, $\nu = 0.3$ and a displacement field

$$u = (c_1 x^2 + c_2 y + c_3 x y^2 + c_4 y^3)\times 10^{-5}$$

$$v = (c_5 y + c_6 x^3 y^2 + c_7 x y + c_8 y^2)\times 10^{-5}$$

where $c_1 = 5.1\ \text{m}^{-1}$, $c_2 = 7.3$, $c_3 = 4.2\ \text{m}^{-2}$, $c_4 = 8.5\ \text{m}^{-2}$, $c_5 = 2.7$, $c_6 = 3.9\ \text{m}^{-4}$, $c_7 = 6.8\ \text{m}^{-1}$ and $c_8 = 5.4\ \text{m}^{-1}$.

(4) (a) Find the rigid body rotation at x = 1, y = 2.

(4) (b) Find the total change in angle of a line element at x = 2, y = 3 where the line element is oriented 20° CCW (counter-clockwise) from the x-direction.

(2) 7. The stress state at a point with respect to an x, y, z coordinate system is given by

$$[\sigma]_{x,y,z} = \begin{bmatrix} 3 & 6 & 0 \\ 6 & 9 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{ MPa.}$$

If you wanted to determine the normal stress on a plane whose outward normal is oriented at angles 40°, 60°, and 66.2° with respect to the x, y and z axes, respectively, which method would you use? (Please circle one).

(a) Eigenvalue method

(b) Mohr's circle

(c) Cosine transformation law

(4) 8. Given a uniform state of plane stress where $\Theta_{xy} = 52.6\mu$ and $\frac{\partial u}{\partial y} = 13.9\mu$, find the change in angle of a line element in the x-direction.

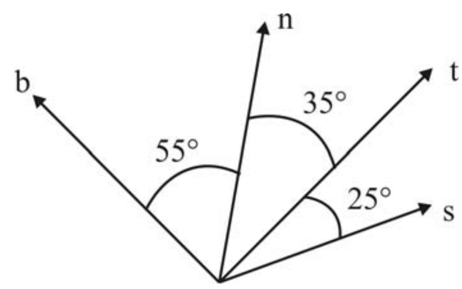
(6) 9. A point in a loaded body is in a state of plane strain. At this point we have

$\varepsilon_s = 68.2\ \mu$

$\varepsilon_t = 32.7\ \mu$

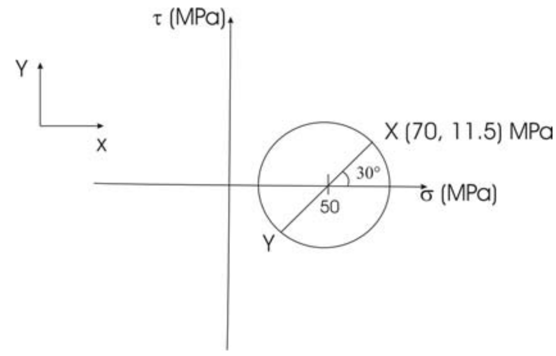
$\varepsilon_n = -34.14\ \mu$

$\varepsilon_b = 59.1\ \mu$



Find γ_{sn}.

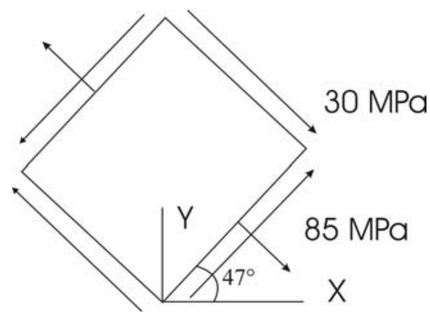
10. A body is in a state of plane stress. The following Mohr's circle represents the stress experienced at a particular point.



(4) (a) Show the stresses corresponding to the x-y coordinate system on a properly oriented element.

(2) (b) If the body is not in a uniform state of plane stress, what conclusions (if any) could you make about the stresses at other points in the body?

(4) 11. A body is loaded in a state of plane stress as shown.



Determine the stresses on a plane perpendicular to the x-axis using the cosine transformation law.

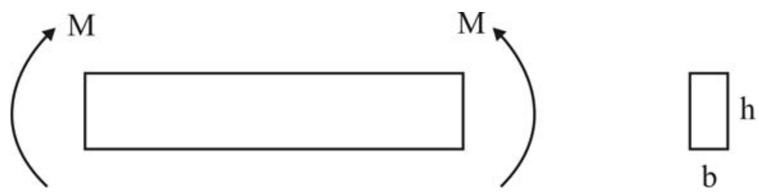
12. The stress state at a point with respect to an x, y coordinate system is given by

$$[\sigma]_{x,y} = \begin{bmatrix} 0 & 25 \\ 25 & 0 \end{bmatrix} \text{ MPa.}$$

(4) (a) Using Mohr's circle what is τ_{max}?

(4) (b) Find σ_1, σ_2 and σ_3.

13. A beam is subjected to a bending moment as shown.



Knowing that for the beam, the Young's modulus in tension, E_t, is twice the Young's modulus in compression, E_c,

(6) (a) find the location of the neutral axis from the bottom of the beam.

(6) (b) Find the formula that gives the bending moment, M, in terms of the curvature of the beam, ρ, as well as E_t, b and h.

(8) 14. In class we derived the formula for $\varepsilon_x = \partial u / \partial x$ which is valid for geometric linearity and therefore deformation where all changes in angle are very small. Keeping the definition of ε_x as the change in length divided by the original length of a line element originally along the x-direction, derive the formula for ε_x in terms of displacement derivatives for the geometrically nonlinear case where angular changes are large.

End of Exam