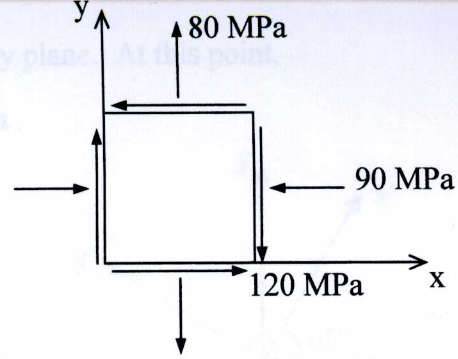
(20) 1. A point in a loaded body is in a state of plane stress as shown. <u>Using the eigenvalue method</u>, determine the in-plane principal stresses and directions. Show your work; i.e., give the quadratic equation for the principal stresses and determine the principal directions using the eigenvalue equations. Show your final answer on a properly oriented element.




$\sigma_{xx} = -90$ MPa

$\sigma_{yy} = 80$ MPa

$\tau_{xy} = \tau_{yx} = -120$ MPa

$$[\sigma]_{xy} = \begin{bmatrix} -90 & -120 \\ -120 & 80 \end{bmatrix} \text{ MPa}$$

20/20

$I_1 = \sigma_{xx} + \sigma_{yy}$

$= -90 \text{ MPa} + 80 \text{ MPa}$

$I_1 = -10$ MPa

$I_2 = \det[\sigma]_{xy} = -21\,600$

$\lambda^2 - I_1\lambda + I_2 = 0$

$\lambda^2 + 10\lambda - 21\,600 = 0$

$\lambda = 142.0544$ MPa or $\lambda = -152.0544$ MPa

6
4
6
4

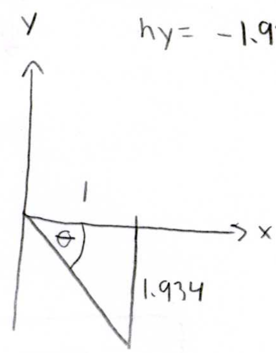
directions

$(\sigma_{xx} - \lambda) n_x + \tau_{xy} n_y = 0$

$(-90 - 142.0544) n_x - 120 n_y = 0$

$n_y = \dfrac{232.0544 n_x}{-120}$, let $n_x = 1$

$n_y = -1.934$

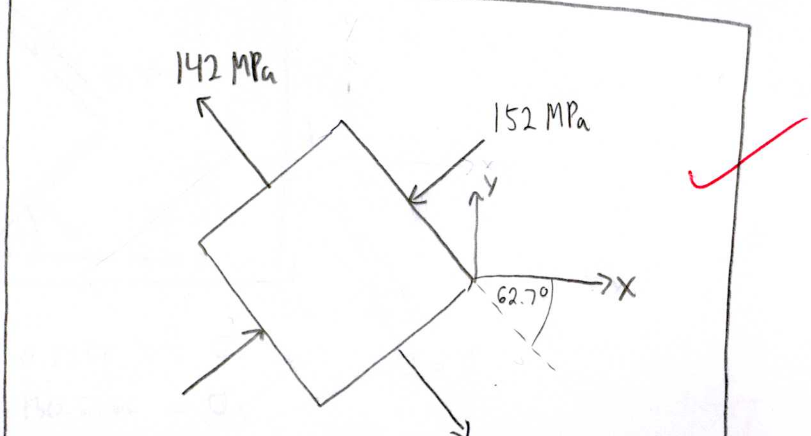



$\theta = \tan^{-1}(1.934)$

$\theta = 62.658° \approx 62.7°$

$\sigma_{max} = 142$ MPa

$\sigma_{min} = -152$ MPa



(20) 2. A point in a loaded body is in a state of plane stress in the x-y plane. At this point,

$\sigma_x = -40$ MPa , $\sigma_y = -60$ MPa , and $\tau_{xy} = -80$ MPa .

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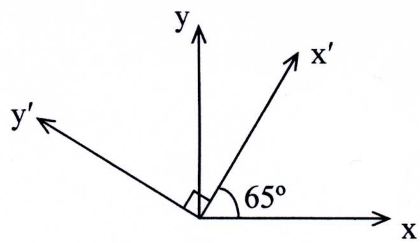
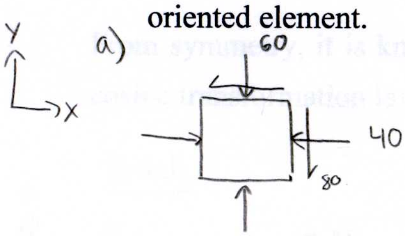



(a) Draw a properly labeled sketch of Mohr's circle and use the circle to determine $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$. Show your answers on a properly oriented sketch of the x'-y' element.

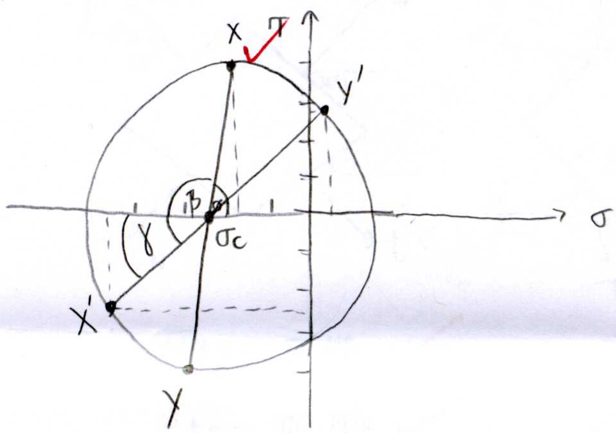
(b) Determine the maximum shearing stress at the point in the loaded body. Show your result on a properly oriented element.

a)




X(-40, 80)

Y(-60, -80)




$\sigma_c = (-50, 0)$ ✓

$\Rightarrow \; r = \sqrt{80^2 + (-50 - -40)^2}$

$r = 80.6226$ ✓

$\phi = \tan^{-1}\left(\frac{80}{10}\right) = 82.875°$ ✓

move X ccw 2(65°) to get X'

$\beta = 130°$

$\gamma = \beta + \phi - 180° = 130° + 82.875° - 180°$

$\gamma = 37.875°$ ✗

$\sigma_{x'} = \sigma_c - r\cos\gamma$

$= -113.64$ MPa ✗

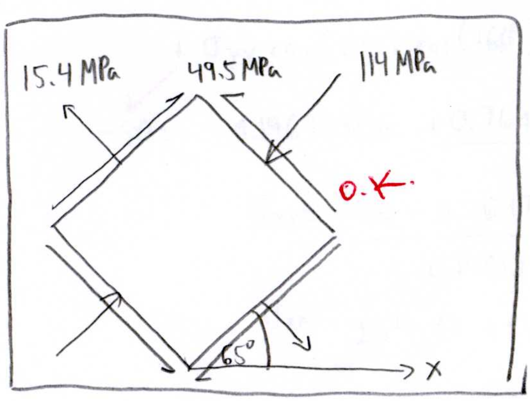
$\sigma_{y'} = \sigma_c + r\cos\gamma$

$\sigma_{y'} = 15.418$ MPa ✗

method O.K.

$\tau_{y'x'} = r\sin\gamma = 49.49$

$\tau_{x'y'} = -r\sin\gamma = -49.49$




O.K.

2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
0 ✓
0 ✓

b)

$\sigma_{max} = \sigma_c + r = 30.6266 = \sigma_1$

$\sigma_{min} = \sigma_c - r = -130.6266 = \sigma_3$

$\therefore \; \sigma_2 = 0$

τ element ?

(20) 3. An element from a loaded body is shown below. The body is in a state of plane stress.

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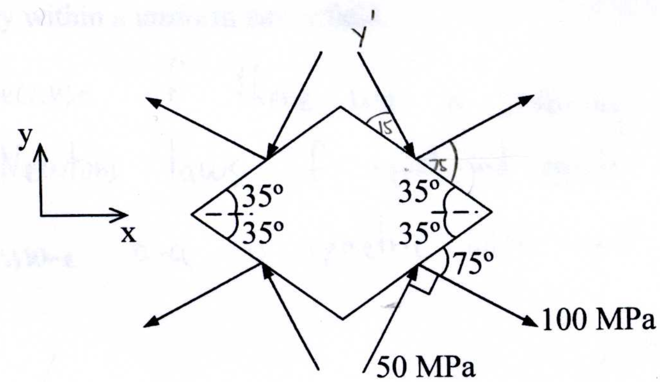
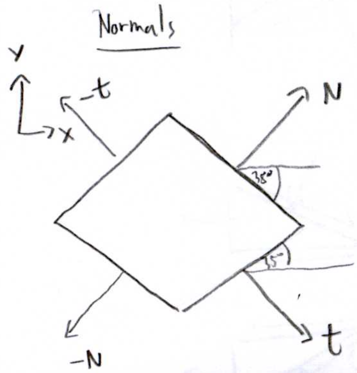
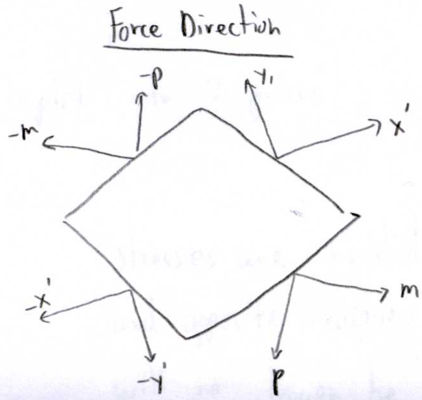
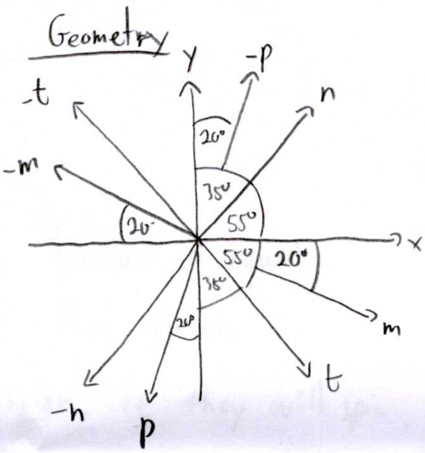
8 ✓
6 ✓
3 ✓




From symmetry, it is known that $\tau_{xy} = 0$. Using the labeled stress components and the cosine transformation law, determine $\sigma_x$ and $\sigma_y$.




$\sigma_{tp} = -50$ MPa ✓

$\sigma_{tm} = 100$ MPa ✓

$\sigma_{tp} = \sigma_{xx}\cos(t,x)\cos(p,x)$
~~$+ \sigma_{xy}\cos(t,x)\cos(p,y)$~~ → 0
~~$+ \sigma_{yx}\cos(t,y)\cos(p,x)$~~ → 0
$+ \sigma_{yy}\cos(t,y)\cos(p,y)$

$\sigma_{tm} = \sigma_{xx}\cos(t,x)\cos(m,x) + \sigma_{yy}\cos(t,y)\cos(m,y)$ because $\tau_{xy} = \tau_{yx} = 0$

$\sigma_{tp} = \sigma_{xx}\cos(55°)\cos(110°) + \sigma_{yy}\cos(145)\cos(160°)$

$\sigma_{tm} = \sigma_{xx}\cos(55°)\cos(20°) + \sigma_{yy}\cos(145)\cos(110)$

$100 = 0.53899\,\sigma_{xx} + 0.28017\,\sigma_{yy}$ ②

✓ $-50 = -0.19617\,\sigma_{xx} + 0.76975\,\sigma_{yy}$ ①

$\sigma_{xx} = \dfrac{-50 - 0.76975\,\sigma_{yy}}{-0.19617}$

$\sigma_{xx} = 254.88 + 3.9239\,\sigma_{yy}$ ③

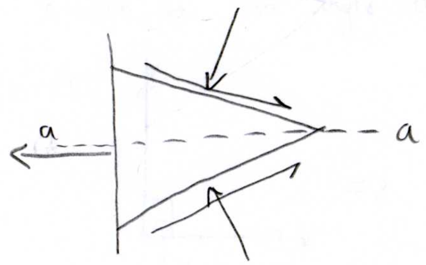
③ → ②

$100 = 137.38 + 2.115\,\sigma_{yy} + 0.28017\,\sigma_{yy}$

(5) 4. **Briefly explain using words and appropriate sketches why the shearing stress is zero on a plane of symmetry within a uniform stress field.**

This because if there was a shearing stress it would violate Newtons laws of equal and opposite forces and equalibrium

ex Assume a-a is symetric with respect to loading and geometry.

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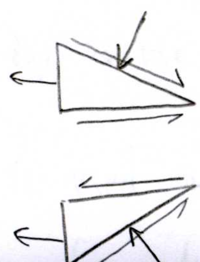
split into 2 pieces



stresses are needed as show because of equal and opposite reactions but now the pieces will no longer be in equalibrium, ie they will spin.

so $\tau$ along a-a must = 0.

(5) 5. **In class, we have discussed "stress" and "strain", each of which is a tensor quantity. Briefly state in words the requirements that any quantity must satisfy in order to be a second order tensor.**

To be a tensor Mohr's circle and the cosine transformation law must work. 2nd order tensors have no magnitude or total stress or strain like vectors.

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While vectors (1st order tensors) are dependant on 1 direction ie $F_x, F_y, F$
2nd order tensors are dependant on 2 directions ie $\sigma_{xx}, \sigma_{xy}, \sigma_{yy}$.
This is because both of the parts of a tensor depend on orientatio

ie $\sigma_{xx} = \frac{F_x}{A_x}$ wheare as 1st order tensor only depend on 1

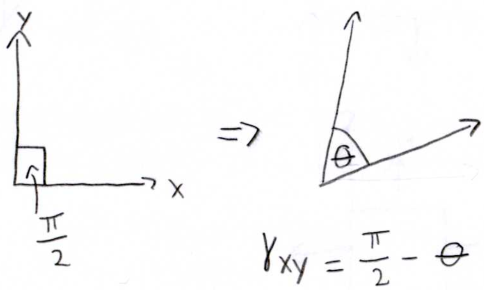
dection $F_x = max$

(10) 6. Using words and sketches, describe the difference between shearing strains $\varepsilon_{xy}$ and $\gamma_{xy}$. As part of your explanation, give the simple mathematical relation between these two quantities. Briefly state why both strains are useful in engineering analysis.

The mathematical relation between $\varepsilon_{xy}$ and $\gamma_{xy}$ is $\varepsilon_{xy} = \frac{\gamma_{xy}}{2}$. ✓

The difference between $\varepsilon_{xy}$ and $\gamma_{xy}$ is that $\gamma_{xy}$ measures the total change in an angle between 2 lines ✓

ie.

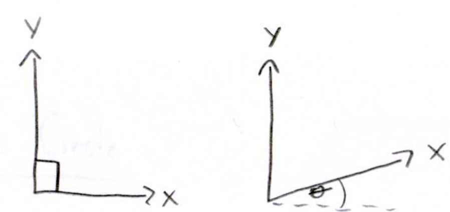



$$\gamma_{xy} = \frac{\pi}{2} - \theta$$

This $\gamma_{xy}$ is use because it has phyical meaning and can be easily measured. ✓

$\varepsilon_{xy}$ is used in all of the tensor math and is the angular change of a line segment in orientation ✓

ie)




$\theta = \varepsilon_{xy}$ ✓

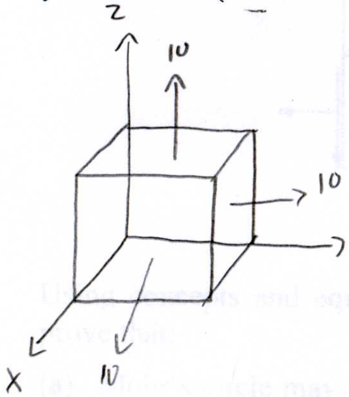
The reason $\gamma_{xy}$ is needed is because $\varepsilon_{xy}$ cannot be measured because its motion can be caused by both solid body rotation and shear strain.

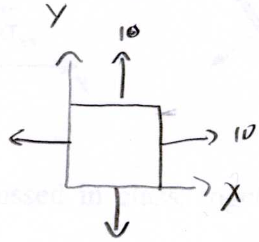
✓

2 ✓
2 ✓
2 ✓
2 ✓
2 ✓

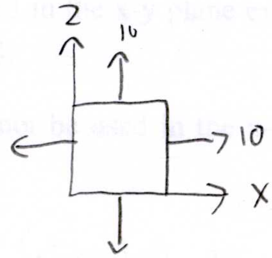
10/10

(10) 7. The stress state at a point in a loaded body is given by $\sigma_{xx} = 10$ MPa, $\sigma_{yy} = 10$ MPa, $\sigma_{zz} = 10$ MPa, $\tau_{xy} = 0$, $\tau_{yz} = 0$, and $\tau_{zx} = 0$. Using a 3-D perspective, sketch the Mohr's circles for this state of stress and thoroughly describe in words the physical significance of your sketch. (Your mark will depend on the completeness of your answer.)

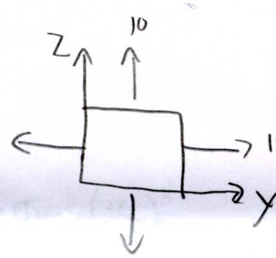



x(10,0)
y(10,0)

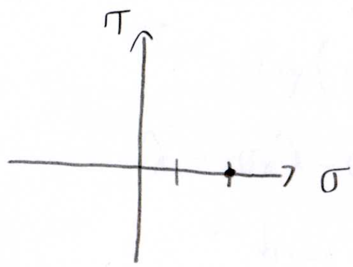



X (10,0)
Z(10,0)




Z(10,0)
y(10,0)

Mohrs Circle




These are principal stresses
$\therefore \tau = 0 = \tau_{xy} = \tau_{xz} = \tau_{zy}$
Every face is a principle face
~~Hdr~~ Hydrostatic state of stress.

Mohrs circle is one point on the σ, τ plane.
This means that this point has only 1 state of stress which means that 1) The loaded body is in a uniform stress field
2) it means this point on the loaded body is along a symetric plane with respect to geometry and loading because it

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