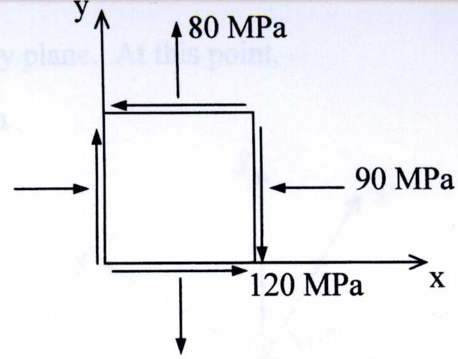
(20) 1. A point in a loaded body is in a state of plane stress as shown. Using the eigenvalue method, determine the in-plane principal stresses and directions. Show your work; i.e., give the quadratic equation for the principal stresses and determine the principal directions using the eigenvalue equations. Show your final answer on a properly oriented element.



$\sigma_{xx} = -90$ MPa

$\sigma_{yy} = 80$ MPa

$\tau_{xy} = \tau_{yx} = -120$ MPa

$$[\sigma]_{xy} = \begin{bmatrix} -90 & -120 \\ -120 & 80 \end{bmatrix} \text{ MPa}$$

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$I_1 = \sigma_{xx} + \sigma_{yy}$

$= -90\text{ MPa} + 80\text{ MPa}$

$I_1 = -10$ MPa

$I_2 = \det[\sigma]_{xy} = -21\,600$

$\lambda^2 - I_1\lambda + I_2 = 0$

$\lambda^2 + 10\lambda - 21\,600 = 0$

$\lambda = 142.0544$ MPa or $\lambda = -152.0544$ MPa

6
4
6
4

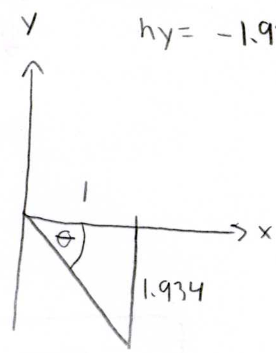
directions

$(\sigma_{xx} - \lambda)n_x + \tau_{xy} n_y = 0$

$(-90 - 142.0544)n_x - 120 n_y = 0$

$n_y = \frac{232.0544 n_x}{-120}$, let $n_x = 1$

$n_y = -1.934$

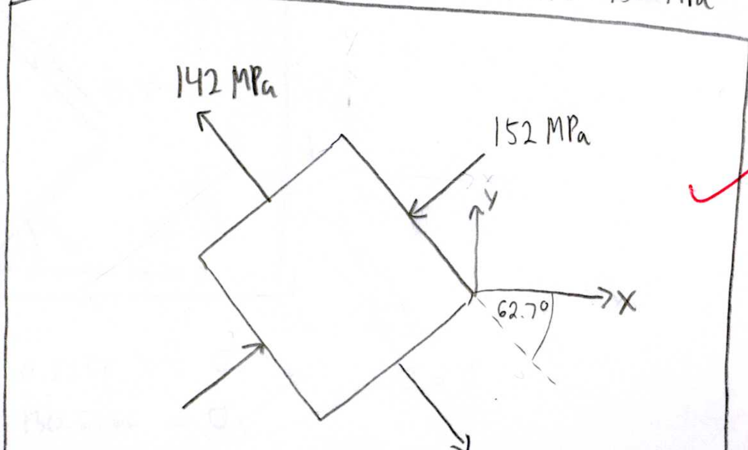


$\theta = \tan^{-1}(1.934)$

$\theta = 62.658° \approx 62.7°$

$\sigma_{max} = 142$ MPa

$\sigma_{min} = -152$ MPa



(20) 2. A point in a loaded body is in a state of plane stress in the x-y plane. At this point,

$$\sigma_x = -40 \text{ MPa}, \quad \sigma_y = -60 \text{ MPa}, \text{ and } \tau_{xy} = -80 \text{ MPa}.$$

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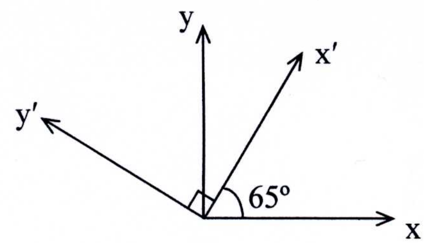
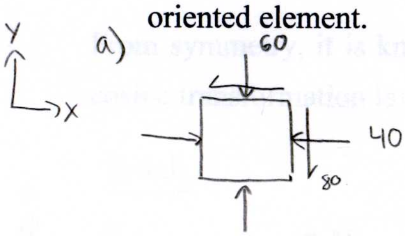


(a) Draw a properly labeled sketch of Mohr's circle and use the circle to determine $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$. Show your answers on a properly oriented sketch of the x'-y' element.

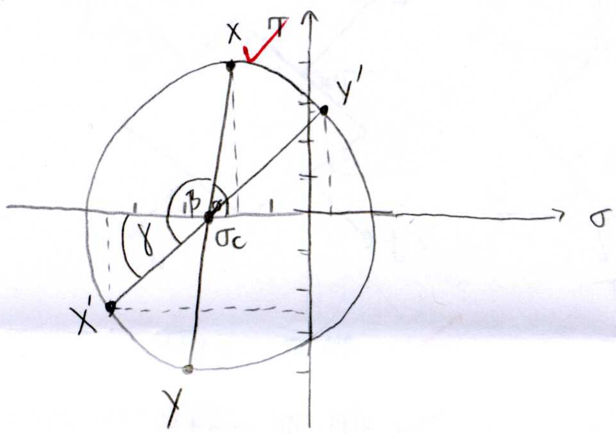
(b) Determine the maximum shearing stress at the point in the loaded body. Show your result on a properly oriented element.

a)



X(-40, 80)

Y(-60, -80)



$\sigma_c = (-50, 0)$ ✓

$\Rightarrow r = \sqrt{80^2 + (-50 - -40)^2}$

$r = 80.6226$ ✓

$\phi = \tan^{-1}\left(\frac{80}{10}\right) = 82.875°$ ✓

move X ccw 2(65°) to get X'

$\beta = 130°$

$\gamma = \beta + \phi - 180° = 130° + 82.875° - 180°$

$\gamma = 37.875°$ X

$\sigma_{x'} = \sigma_c - r\cos\gamma$

$= -113.64$ MPa X

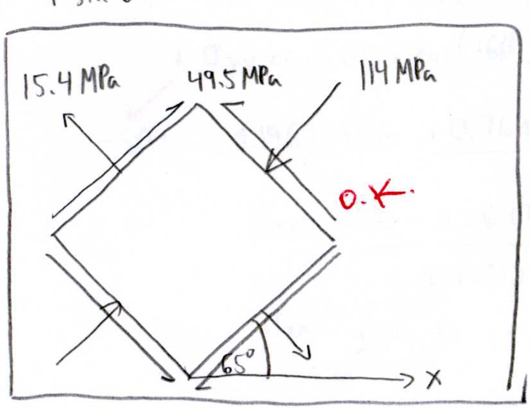
$\sigma_{y'} = \sigma_c + r\cos\gamma$

method O.K.

$\sigma_{y'} = 15.418$ MPa X

$\tau_{y'x'} = r\sin\gamma = 49.49$

$\tau_{x'y'} = -r\sin\gamma = -49.49$



O.K.

2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
2 ✓
0 ✓
0 ✓

b)

$\sigma_{max} = \sigma_c + r = 30.6266 = \sigma_1$ $\therefore \sigma_2 = 0$

$\sigma_{min} = \sigma_c - r = -130.6266 = \sigma_3$

τ element?

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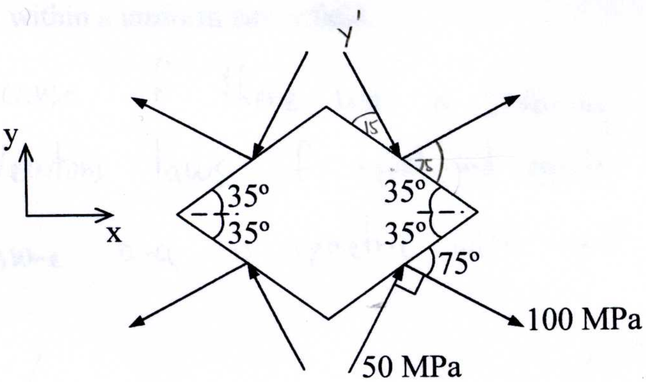
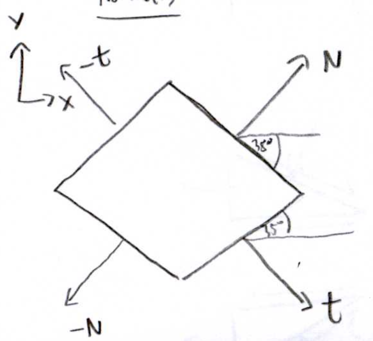
8
6
3

(20) 3. An element from a loaded body is shown below. The body is in a state of plane stress.

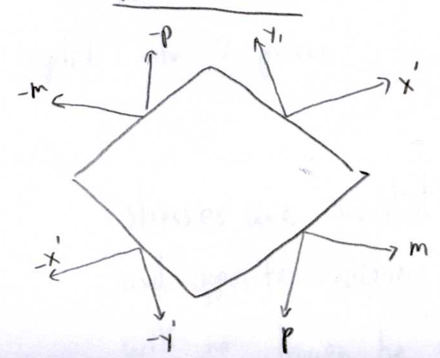


From symmetry, it is known that $\tau_{xy} = 0$. Using the labeled stress components and the cosine transformation law, determine σ_x and σ_y.

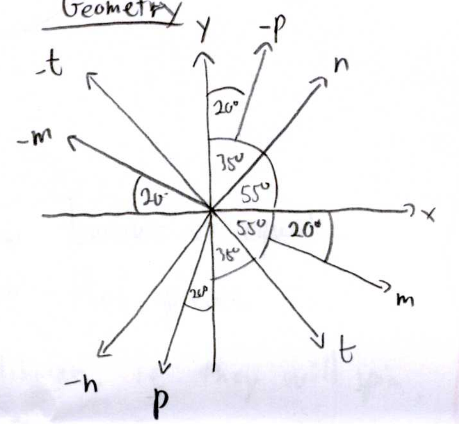
Normals



Force Direction



Geometry



$\sigma_{tp} = -50$ MPa

$\sigma_{tm} = 100$ MPa

$\sigma_{tp} = \sigma_{xx}\cos(t,x)\cos(p,x) + \cancel{\sigma_{xy}\cos(t,x)\cos(p,y)} \to 0 + \cancel{\sigma_{yx}\cos(t,y)\cos(p,x)} \to 0 + \sigma_{yy}\cos(t,y)\cos(p,y)$

$\sigma_{tm} = \sigma_{xx}\cos(t,x)\cos(m,x) + \sigma_{yy}\cos(t,y)\cos(m,y)$ because $\tau_{xy} = \tau_{yx} =$

$\sigma_{tm} = \sigma_{xx}\cos(55°)\cos(20°) + \sigma_{yy}\cos(145)\cos(110)$

$\sigma_{tp} = \sigma_{xx}\cos(55°)\cos(110°) + \sigma_{yy}\cos(145)\cos(160°)$

$100 = 0.53899\,\sigma_{xx} + 0.28017\,\sigma_{yy}$ ②

$-50 = -0.19617\,\sigma_{xx} + 0.76975\,\sigma_{yy}$ ①

$$\sigma_{xx} = \frac{-50 - 0.76975\,\sigma_{yy}}{-0.19617}$$

$\sigma_{xx} = 254.88 + 3.9239\,\sigma_{yy}$ ③

③ → ②

$100 = 137.38 + 2.115\,\sigma_{yy} + 0.28017\,\sigma_{yy}$