**UNIVERSITY OF SASKATCHEWAN**
**DEPARTMENT OF MECHANICAL ENGINEERING**
**ME 316.3 – SYSTEM DYNAMICS AND VIBRATIONS**
**FINAL EXAMINATION – DECEMBER 9, 2004**

INSTRUCTOR: CHRIS WJ ZHANG

**3** HOURS (9:00 -12:00 A.M.)
CLOSED BOOK, CALCULATORS PERMITTED.
SHOW YOUR WORK. THERE **3** QUESTIONS
ANSWER **ALL** QUESTIONS.

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## FORMULAE:

$$\vec{V}_B = \vec{V}_A + \frac{dr_{B/A}}{dt}\hat{u}_{B/A} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \frac{d^2 r_{B/A}}{dt}\vec{u}_{B/A} + 2\vec{\omega}_{AB} \times \frac{dr_{B/A}}{dt}\vec{u}_{B/A}$$
$$+ \vec{\alpha}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})$$

$\frac{d\vec{A}}{dt} = \vec{\omega} \times \vec{A}$, if the magnitude of vector A is constant.

**More formulae can be found in the appendix at the end of this paper**

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**Question 1:**

Figure 1 shows a system which consists of the rod CD (weight: $m_1$=1 kg) and the disk (weight: $m_2$=2 kg) which is attached at the point D. The disk is further laid against the rod AB. The rod CD is joined with the rod AB by a pin at point C such that the CD and the disk can only rotate about the pin axis relative to the rod AB. The weight of the rod AB is negligible, and it is supported at B by a smooth journal bearing and at A by a smooth thrust bearing. At the instant shown, the rod CD lies in the Y-Z coordinate plane, and the pin axis at the point C is along the X-axis. Furthermore, the rod AB rotates about the Z-axis (Figure 1), and there is a torque (T=10 Nm) applied on the rod AB (Figure 1). The angular velocity of the system with respect to the Z-axis (ω=1 rad/s). For the instant shown, determine the following:

a. The center of gravity of the system expressed in the X-Y-Z coordinate system. [5]
b. The moment of inertia of the system with respect to the Z axis. [10]
c. The angular acceleration of the system about the Z-axis. [10]
d. The acceleration at the center of the gravity of the system. [5]

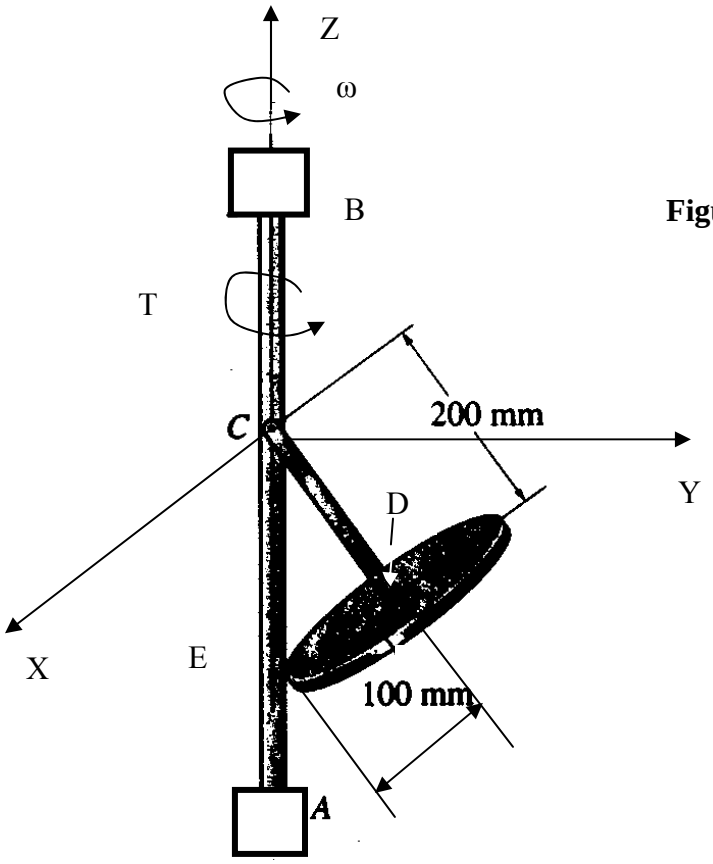


**Figure 1**

**Question 2:**

The rod assembly (see Figure 2) is supported by a journal bearing at point D and by a thrust bearing at point C. The rods (AB and CD) have a mass of 0.75 kg / m. The rods have a constant angular acceleration $\alpha$=200 rad / $s^2$. There is a force F applied horizontally on the rod CD, and F=10 N. At the instant shown, the rod AB is vertically laid in the Y-Z plane, and the angular velocity $\omega$=8 rad / s. For the instant shown determine the following:

a. The angular velocity of the rods (with respect to the X-Y-Z coordinate system). [5]
b. The angular acceleration of the rods (with respect to the X-Y-Z coordinate system). [5]
c. The force diagram which shows the components of reactions (C,D). [10]
d. The torque on the rod DC. [5]
e. The components of reactions at the support (C,D). [10]
f. The joint force between the rod AB and the rod CD. [10]
g. The kinetic energy of the rod assembly. [5]

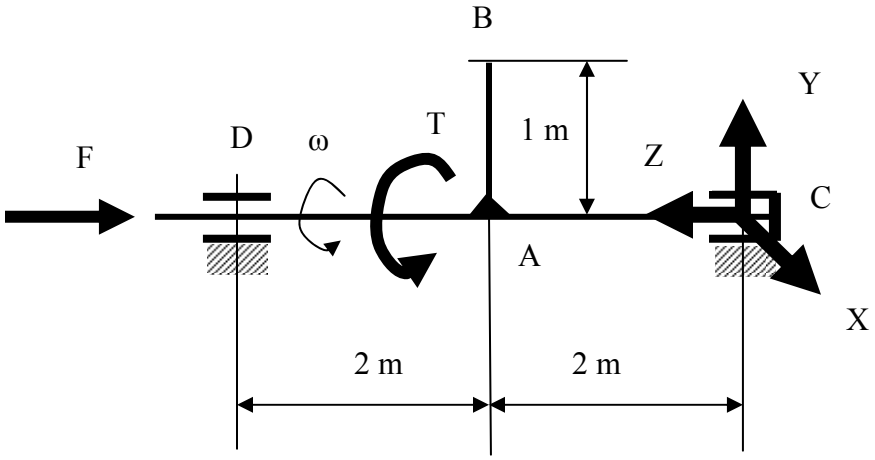


**Figure 2**

3. The electric motor turns an eccentric flywheel which is equivalent to an unbalanced 0.25 –lb weight located 10 in from the axis of rotation (see Figure 3). The total weight of the motor and the flywheel (together) is 150 lb. Neglect the mass of the beam. If the static deflection of the beam is 1 in due to the weight of the motor and the flywheel determine:

   a. The stiffness of the system. [3]
   b. The natural frequency of the system. [5]
   c. The angular velocity of the flywheel at which the resonance occurs; [3]
   d. The critical damping coefficient $C_c$ [3]

Further,

   e. Explain two ways to change the natural frequency of the system. [6]

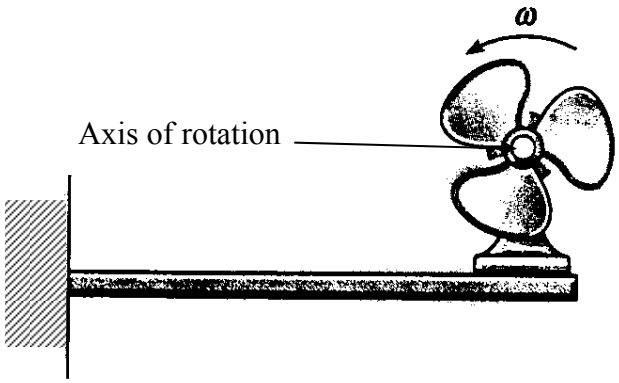


**Figure 3**

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