Student Number: ____________________ Name: ______________________________

M E 321.3 – ENGINEERING ANALYSIS II

Department of Mechanical Engineering
University of Saskatchewan

Midterm Exam

Wednesday, October 18, 2006, 3:00 – 5:00pm

Instructor: Prof. FangXiang Wu

PLEASE READ CAREFULLY:

This exam has 4 pages. Some useful formula may be found on the last page of the exam. The exam is closed book. No calculators or other aids are permitted. There are 4 problems on the exam. Attempt all 4 problems. Please ensure that your answers are clear and legible. You must return the question sheets at the end of the exam.

Problem 1 (10 points)

a. For the following partial differential equation, identify the dependent and independent variables, and specify if the equation is homogeneous or non-homogeneous. What is the order of the equation? Is the equation linear or nonlinear? Is the equation hyperbolic, elliptic, or parabolic?

$$\frac{\partial z}{\partial x}+\frac{1}{x}\frac{\partial z}{\partial y}+4z=\frac{\partial^2 z}{\partial x^2}+\frac{1}{x^2}\frac{\partial^2 z}{\partial y^2}-\frac{2}{x}\frac{\partial^2 z}{\partial x\partial y}+2x-3y \text{ , where } x>0$$

b. What is the superposition principle for linear homogeneous PDEs?

Problem 2 (30 points)

Let $f(x)$ be a periodical function with the fundamental period $T=2$ and be defined on [-1, 1) as follows:

$$f(x)=x \text{ , on the interval } -1\le x<1$$

a. Compute the Fourier series expansion of the above function on the given interval.

b. Provide a clearly labeled sketch of the function over two fundamental periods.

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c. Use the result of a) to show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

d. Use the result of a) and Parseval's Theorem to show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

<u>Problem 3</u> (30 points)

Of interest is the steady-state temperature distribution, $T(r,\theta)$, in the two-dimensional quarter-disk shown in Figure 1 below. The quarter-disk is of radius, a, and there is no heat addition. The temperature about the outer circumference of the disk is given by a piecewise function, $f(\theta)$, which is defined below.

$$f(\theta) = \begin{cases} 10°\text{C}, & 0 \le \theta < \pi/4 \\ 5°\text{C}, & \pi/4 \le \theta \le \pi/2 \end{cases}$$

The remaining two edges are insulated, as shown in Figure 1. The steady-state temperature distribution, $T(r,\theta)$, is found by solving Laplace's equation in polar coordinates,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0.$$

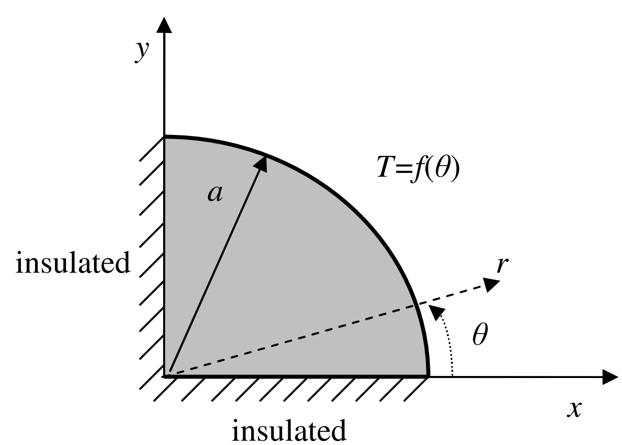

Figure 1

a. What are the boundary and physical conditions that define this problem?

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b. Using the separation-of-variables technique, find an analytical expression for the steady-state temperature distribution, $T(r,\theta)$, on the quarter-disk.

Problem 4 (30 points)

Consider the unsteady temperature distribution in the metal rod (c^2 = 3 m^2/s) of length L = 6 m, shown in Figure 2, where two ends are maintained at constant temperatures -6 [°C] and 6 [°C], respectively. The metal rod may be treated as a one-dimensional, unsteady heat conduction problem, with no heat addition. The initial temperature distribution in the rod is given by the function $f(x)$ below.

$$f(x) = 2x\,[°\text{C}],\ 0 < x < 6\,\text{m}$$

a. Write out the partial differential equation that must be solved, as well as the boundary conditions and initial condition that define the problem.

b. Use the separation-of-variables technique to find an analytical expression for the unsteady temperature distribution, $T(x, t)$, in the metal rod.

c. Using a *clearly labeled* sketch, plot the temperature distribution along the length of the bar. Using several curves, show how the temperature distribution varies with time. Your plot should include the initial temperature distribution, the steady-state solution, and several intermediate time steps.

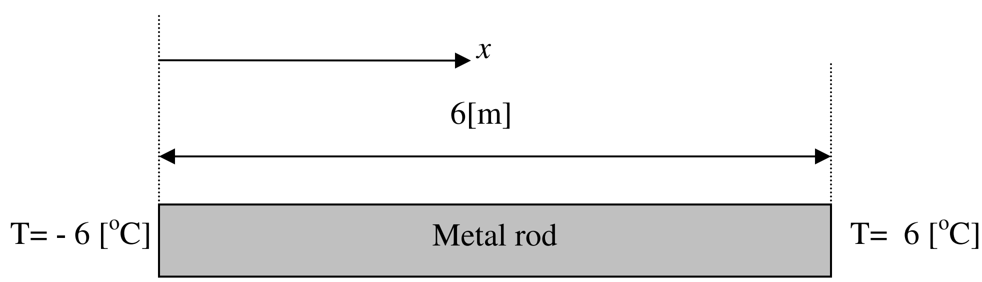

Figure 2

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Some Useful Information

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + C$$

$$\int x\sin(ax)dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + C$$

$$\int x\cos(ax)dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax) + C$$

$$\int x^2 \sin(ax)dx = -\frac{x^2}{a}\cos(ax) + \frac{2x}{a^2}\sin(ax) + \frac{2}{a^3}\cos(ax) + C$$

$$\int x^2 \cos(ax)dx = \frac{x^2}{a}\sin(ax) + \frac{2x}{a^2}\cos(ax) - \frac{2}{a^3}\sin(ax) + C$$

$$\int x^n \sin(ax)dx = -\frac{x^n}{a}\cos(ax) + \frac{n}{a}\int x^{n-1}\cos(ax)dx$$

$$\int x^n \cos(ax)dx = \frac{x^n}{a}\sin(ax) - \frac{n}{a}\int x^{n-1}\sin(ax)dx$$

$$\sinh(ax) = \frac{e^{ax} - e^{-ax}}{2}$$

$$\cosh(ax) = \frac{e^{ax} + e^{-ax}}{2}$$