Student Number: ____________________ Name:____________________________

M E 321.3 – ENGINEERING ANALYSIS II

Department of Mechanical Engineering
University of Saskatchewan

Midterm Exam

Wednesday, October 21, 2009, 3:30 pm– 5:30pm

Instructor: Prof. FangXiang Wu

PLEASE READ CAREFULLY:

This exam has 4 problems on 9 pages (including answer spaces and a formula sheet). The exam is closed book. However, you may bring the help sheet made by yourself on one page of a letter-sized paper and a scientific calculator. Attempt all 4 problems. Please ensure that your answers are clear and legible.

Question	Total Marks	Score
1	15	14
2	25	~~18~~ 18
3	30	30
4	30	24
<u>TOTAL</u>	**100**	~~90~~ 86

I declare that I am the person named, and that I am formally registered as a student in ME 321.

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Signature **Date**

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Problem 1 (15 points)

a. For the following partial differential equation, identify the dependent and independent variables, and specify if the equation is homogeneous or non-homogeneous. What is the order of the equation? Is the equation linear or nonlinear? Is the equation hyperbolic, elliptic, or parabolic?

$$\frac{\partial z}{\partial r}+\frac{1}{r}\frac{\partial z}{\partial \theta}+4z=\frac{\partial^2 z}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2}+\frac{1}{r}\frac{\partial^2 z}{\partial r \partial \theta}+2r-3\theta \text{ where } r>0$$

b. What are the definitions of an "odd function" and an "even function"?

a) z is the dependant variable

r and θ are the independant variables

The equation is non-homogeneous.

The equation is a 2^{nd} order equation

The equation is linear.

$A=1$ $B=\frac{1}{r}$ $C=\frac{1}{r^2}$

$\Delta = B^2 - 4AC$

$\Delta = \frac{1}{r^2} - \frac{4}{r^2} = \frac{-3}{r^2}$

$\Delta = \frac{-3}{r^2}$ is always negetive $\therefore$ ✓

$\boxed{\text{The Equation is Elliptic}}$

b) an odd function symetric about the the +x,+y quadrent and the -x,-y quadrent. and as a result if f(x) is odd then $f(-x) = -f(x)$ and $\int_{-L}^{L} f(x)_{odd}\, dx = 0$

an even function is symetric about the y axis and as a result if f(x) is even then $f(-x) = f(x)$ and $\int_{-L}^{L} f(x)_{even}\, dx = 2\int_{0}^{L} f(x)_{even}\, dx$

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Problem 2 (25 points)
Consider the following piecewise function $f(x)$ defined on the interval [-2, 2).

$$f(x) = \begin{cases} x, & -2 \le x < 0 \\ 1, & 0 \le x < 2 \end{cases}$$

a. Expand the function in a Fourier series with period $T = 2L = 4$.
b. Provide a clearly labeled sketch of the Fourier series obtained in (a) over an interval with the length of 8.
c. Use the Fourier series obtained in a) to verify that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)}$$

a) $f(x) \simeq \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-2}^{0} x\,dx + \frac{1}{2}\int_{0}^{2} dx = \frac{1}{2}\left[-\frac{1}{2}(-2)^2 + 2\right]$$

$$a_0 = 0$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{n\pi x}{2}\right)dx = \frac{1}{2}\int_{-2}^{0} x\cos\left(\frac{n\pi x}{2}\right)dx + \frac{1}{2}\int_{0}^{2}\cos\left(\frac{n\pi x}{2}\right)dx$$

$$a_n = \frac{1}{2}\left(\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) + \frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{-2}^{0} + \frac{1}{2}\left(\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{0}^{2}$$

$$a_n = \frac{1}{2}\left(\frac{4}{n^2\pi^2} - \frac{4}{n^2\pi^2}\cos(-n\pi) + \cancel{\frac{4}{n\pi}\sin(-n\pi)}\right) + \cancel{\frac{1}{2}\left(\frac{2}{n\pi}\sin(n\pi) - 0\right)}$$

$$a_n = \frac{2}{n^2\pi^2} - \frac{2(-1)^n}{n^2\pi^2}$$

$$a_n = \frac{2}{n^2\pi^2}\left[1 - (-1)^n\right]$$ ✓

$$b_n = \frac{1}{2}\int_{-2}^{0} x\sin\left(\frac{n\pi x}{2}\right)dx + \frac{1}{2}\int_{0}^{2}\sin\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \frac{1}{2}\left(\frac{4}{n^2\pi^2}\cos\left(\frac{n\pi x}{2}\right) - \frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{-2}^{0} + \frac{1}{2}\left(\frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\right)\Bigg|_{0}^{2}$$

$$b_n = \frac{1}{2}\left(\frac{4}{n^2\pi^2} - \frac{4}{n^3\pi^2}\cos(-n\pi) + \frac{4}{n\pi}\sin(-n\pi)\right) \Bigg| +$$

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