# ME 327 – Heat Transfer

October 23, 2009 — Mid Term Exam — 120 minutes

**75 Points Total – 5 questions, each worth 15 points**

Open Book Exam – students are permitted to bring their course notes, assignments and the course text (**or** one other heat transfer book of their choice) into the exam. Other materials are not permitted. Programmable calculators are permitted in exams. Cell phones, PDA's, computers, and tape, CD and digital music players are not permitted.

For numerical questions – please show all work in the space provided.

For descriptive questions – please answer each question in a concise and clear fashion in the space provided using proper spelling and grammar.

**Question 1 (3 points each, 15 points total)** 15

a) What are the three modes of heat transfer? Which mode of heat transfer would be of greatest importance in space applications?

3

The three modes of heat transfer include conduction, convection and radiation. Radiation is of greatest importance in space applications because it can transfer heat through a vacuum.

b) Briefly explain the difference between the thermal energy storage and thermal energy generation terms in an energy balance. What is an example of thermal energy generation?

3

Thermal energy storage is energy stored in the control volume either by increasing its temperature or by causing a phase change. Thermal energy generation is when energy of another kind is converted to thermal energy inside a control volume.
Heat generation from a current carrying wire, $P = I^2R$, is thermal energy generation.

c) What is the maximum thickness of copper plate that can be treated using the lumped heat capacity method if the plate is heated on one side by a gas stream that is at 800°C (convection heat transfer coefficient of 20 W/m$^2$·°C) and the other side is perfectly insulated?

3

From table A.1 for copper $k = 401 \frac{W}{mK}$

$B_i < 0.1$ for lumped capacitance.

$$\frac{hV}{A_s k} < 0.1$$

$$\frac{20 A_s L}{A_s 401} < 0.1$$

$$\boxed{L < 2.005 \text{ m}}$$

$\therefore L_{max} = 2.004 \text{m}$

d) Is the heat transfer rate constant in a cylinder under steady state conditions in which there is no thermal energy generation? How about the heat flux?

3

The heat transfer rate is constant because the temperature of the cylinder is not changing. The heat flux is not constant because

e) The rate of heat transfer from a camp fire is significantly higher than the rate of heat transfer from a candle. Briefly explain why both fires can cause a skin burn in approximately the same length of time.

3

Both a camp fire and a candle have similar heat fluxes although they have different rates of heat transfer. This is due to the physical difference in the size of the fire. They would burn you in about the same time because they have similar heat fluxes, but the burn from the candle would be smaller.

**Question 2 (5 points each, 15 points total)** 10.5 Q2

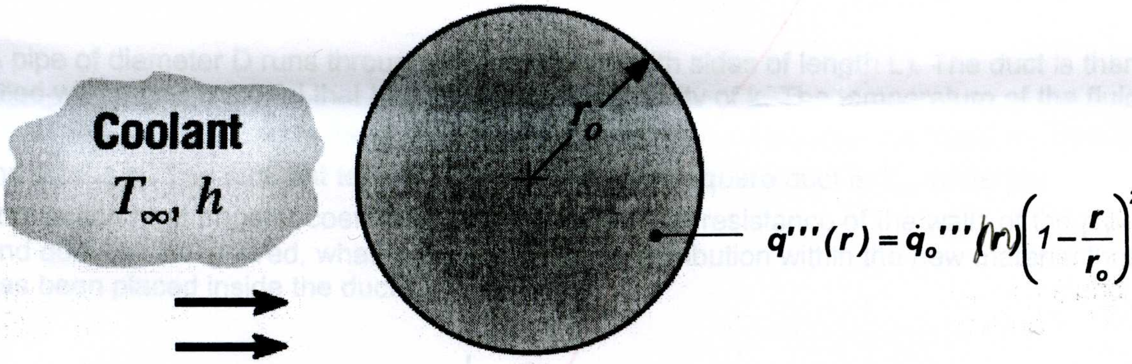
Write the appropriate form of the Fourier field equation, and the boundary and initial conditions that you would use to solve your differential equation for each of the following situations. Clearly state the assumptions that you have used to develop your differential equation, and boundary and initial conditions. **You do <u>not</u> need to solve the differential equation.**

a) Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy according to the following equation.

$$\dot{q}'''(r) = \dot{q}_o'''\left(1-\frac{r}{r_o}\right)^2$$

where $\dot{q}_o'''$ is a constant and $r_o$ is the outer radius of the container. To maintain thermal equilibrium, the container is placed in a coolant that is at a temperature of $T_\infty$, and the convection heat transfer coefficient is estimated to be h. Determine the temperature distribution within the container as a function of the radius.

3.5




Assume - 1-D heat transfer in r
- steady state
- constant thermal properties

$$\frac{1}{r^2} k \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \dot{q}_o'''\left(1-\frac{r}{r_o}\right)^2 = 0$$ ✓ 2/2

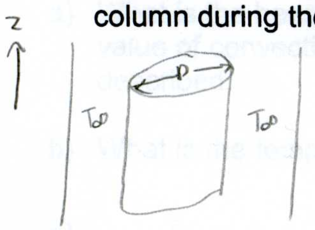
BCs

$$-k \left.\frac{dT}{dr}\right|_{r=r_o} = h\left[T(r=r_o) - T_\infty\right]$$ ✓ 1.5/3

$$\left.\frac{dT}{dr}\right|_{r=0} = \dot{q}_o'''$$ X

that the curved outer surface of the column is exposed to the hot gases. This outer surface has an emissivity of ε. The column is initially at a temperature of $T_i$ when the test begins. The furnace is controlled so that the temperature of the hot gases, $T_\infty$, follows a specified temperature-time curve. Assume that the walls of the furnace are at the same temperature as the hot gases and that the convection heat transfer is h. To increase the accuracy of your solution, you decide to incorporate temperature dependent properties of the concrete. Determine an equation that can be used to calculate the temperature distribution within the column during the first 10 minutes of this test.

5




Assume – $T_\infty = T_{sur}$

– $T_\infty$ is constant along z axis.

– 1-D heat transfer in r

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = \rho C_p \frac{\partial T}{\partial t}$$

BCs

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$$

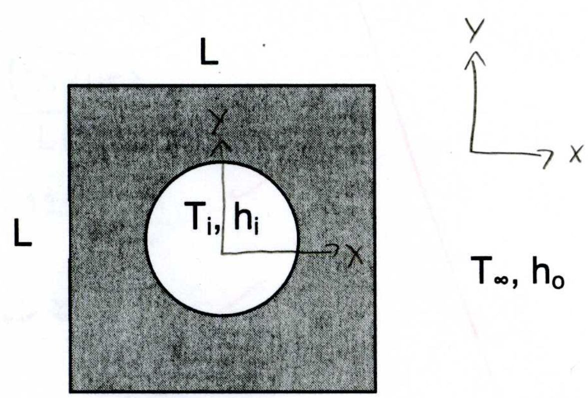
Initial condition

$$T(r, t=0) = T_i$$

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=\frac{D}{2}} = h\left[T_{(r=\frac{D}{2},t)} - T_\infty\right] + \varepsilon\sigma\left(T^4_{(r=\frac{D}{2},t)} - T_\infty^4\right)$$

c) A pipe of diameter D runs through a square duct (with sides of length L). The duct is then filled with a new material that has a thermal conductivity of k. The temperature of the *fluid* inside the pipe is $T_i$, while the convection heat transfer coefficient on the inside surface of the pipe is $h_i$. The ambient temperature outside of the square duct is $T_\infty$, while the convection heat transfer coefficient is $h_o$. If the thermal resistance of the walls of the pipe and duct can be ignored, what is the temperature distribution within the new material that has been placed inside the duct?

2




Assume - 2-D heat transfer

- steady state

fluid

for $\sqrt{x^2+y^2} \le \frac{D}{2}$ $\quad \frac{1}{r}\frac{\partial}{\partial r}\left(k_i r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k_i\frac{\partial T}{\partial \phi}\right) = 0$

for $-L \le x \le L$ $\quad \frac{\partial}{\partial}\left(k_b\frac{\partial T}{\partial}\right) + \frac{\partial}{\partial}\left(k_b\frac{\partial T}{\partial}\right) = 0$

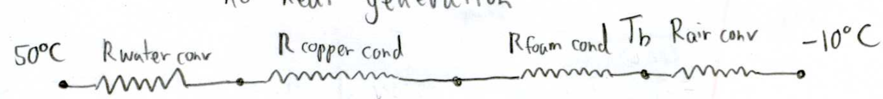
2/

Water, at a temperature of 50°C, flows through a copper pipe. The outer diameter of the pipe is 90 mm and the wall thickness of the pipe is 2.5 mm. The convection heat transfer coefficient on the inside surface of the pipe is 200 W/m$^2$·°C. In order to reduce heat losses, a 10 mm thick layer of rigid foam rubber pipe insulation is wrapped around the pipe. The ambient temperature outside the pipe is -10°C, and there is no wind.

a) What is the heat loss per unit length through the insulated pipe? Assume an appropriate value of convection heat transfer coefficient on the outside of the pipe for the conditions described. (10 points)

b) What is the temperature on the outside surface of the insulation? (5 points)

a) Assume - 1-D heat transfer in r
- steady state
- no heat generation




$$R_{water\ conv} = \frac{1}{hA} = \frac{1}{200\frac{W}{m^2K}\,\pi(0.085m)}$$

$R_{water\ conv} = 0.018724\ \frac{mK}{W}$ ✓

from table A.1 for copper at ≈ 300K $k = 401\ \frac{W}{mK}$ ✓

" " A.3 for foam at ≈ 300K $k = 0.026\ \frac{W}{mK}$ ✓

$$R_{copper\ cond} = \frac{\ln(r_2/r_1)}{2\pi L k} = \frac{\ln(45/42.5)}{2\pi L\ 401}$$

$R_{copper\ cond} = 2.2686\ \frac{mK}{W}$ $2.27(10)^{-5}$

$$R_{foam\ cond} = \frac{\ln(55/45)}{2\pi L(0.026)}$$

$R_{foam\ cond} = 1.2284\ \frac{mK}{W}$ ✓

a) 8/10

b) 5/5

Assume $h_{air} \approx 10\frac{W}{m^2K}$ ✓

$$R_{air\ conv} = \frac{1}{hA} = \frac{1}{10\frac{W}{m^2K}\,\pi(0.11m)}$$

$R_{air\ conv} = 0.2894\ \frac{mK}{W}$ ✓

OK for your #'s

$R_T = \Sigma R = 3.805\frac{mK}{W}$

$$q_{loss\ per\ L} = \frac{\Delta T}{3.805\frac{mK}{W}} = \frac{-60°C}{3.805\frac{mK}{W}} = \boxed{-15.769\ \frac{W}{m}}$$ out of pipe due to - sign