Name: ______________________________

Student No.: ______________________________

M E 335.3 – FLUID MECHANICS II

Department of Mechanical Engineering
University of Saskatchewan

Final Exam

Monday, April 21, 2003, 9:00 a.m. – 12:00 p.m.

Instructor: Professor David Sumner

PLEASE READ CAREFULLY:

This exam has 7 pages. This exam is closed book. You are permitted to use your own calculator; no other aids are permitted. There are 6 problems on the exam. Each problem is of equal value. Attempt all 6 problems. Please ensure that your answers are clear and legible.

Some helpful information is found on the last page of the exam.

You must return the exam question sheets with your exam booklets.

Problem 1

A laminar boundary layer on a smooth flat plate is approximated by a linear velocity profile given by the following equation, for freestream velocity, U_∞, and boundary layer thickness, δ. There is no pressure gradient, and the flow is two-dimensional, steady, and incompressible.

$$\frac{u}{U_\infty} = \frac{y}{\delta}$$

a. In the absence of a pressure gradient, what five boundary conditions should be satisfied for any laminar boundary layer velocity profile? Be sure to provide a short description or definition for each of the five conditions.

b. For the linear velocity profile given above, which of the five conditions are satisfied?

c. Using the Kármán integral momentum equation, obtain an expression for the boundary layer thickness, $\delta(x)$, as a function of x only, for the above linear velocity profile.

d. For a flat plate of length L and width W, show that the drag coefficient can be found simply by evaluating the momentum thickness at the trailing edge, according to the following expression.

$$C_D = \frac{2\theta(L)}{L}$$

Problem 2

The power, P, produced by a wind turbine is *primarily* a function of the air density, ρ, the wind speed, V, and the turbine blade radius, R. *Secondary* variables influencing the power are the air viscosity, μ, the rotational speed, ω, and the mass moment of inertia, I.

a. Use dimensional analysis to develop a functional relationship for the power, P, using all the parameters listed above. Based on the information provided in the problem above, justify your choice of repeating parameters.

b. Provide a short physical interpretation or description for each of the four dimensionless groups from (a).

A prototype wind turbine will encounter a wind speed of 30 km/h in air at standard atmospheric conditions. A 1/5-scale, geometrically similar model of the wind turbine is to be tested in a large water tunnel, where the water is at 20°C. The fluid properties are provided on the last page of the exam.

c. For complete dynamic similarity between the model and prototype, what should be the flow speed in the water tunnel? What should be the ratio of rotational speeds between the model and prototype?

Problem 3

The following velocity field represents a steady, incompressible, two-dimensional flow of a Newtonian fluid. The fluid has density, ρ, viscosity, μ, thermal conductivity, k, and specific heat, c. Gravity may be neglected.

$$u = x + y \qquad v = x - y \qquad w = 0$$

a. Show that the flow is incompressible.

b. Find the local acceleration of the fluid.

c. Find the convective acceleration of the fluid.

d. Use the Navier-Stokes equations to find the pressure field, $P(x,y)$.

e. Consider an elemental control volume of fluid, with dimensions dx and dy and unit depth, as shown in Figure 1 below. For the flow field given above, evaluate all the shear stress components acting on this elemental area, and indicate their direction (if applicable) on a sketch of the elemental area.

f. Based on your analysis of the flow in (a) through (e), explain whether this flow can be considered inviscid.

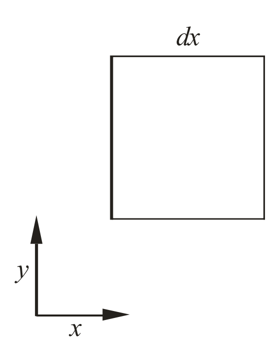


Figure 1

Problem 4

Consider the steady, laminar, incompressible, viscous, axisymmetric, fully developed Newtonian fluid flow in the circular pipe shown in Figure 2 below. The pipe has a radius R. The fluid falls under the action of gravity. The pipe wall moves upwards at a constant velocity, V, countering the action of gravity. The flow is purely axial, with $u_z = u_z(r)$ only, and $u_\theta = u_r = 0$. The fluid has a density, ρ, and a viscosity, μ, and the pressure in the fluid is constant.

a. Specify the boundary condition(s) for the flow velocity, $u_z(r)$.

b. Use the Navier-Stokes equations to find an expression for the velocity profile, $u_z(r)$.

c. Compute the volume flow rate, Q. What upward pipe-wall velocity V is needed for zero volume flow rate?

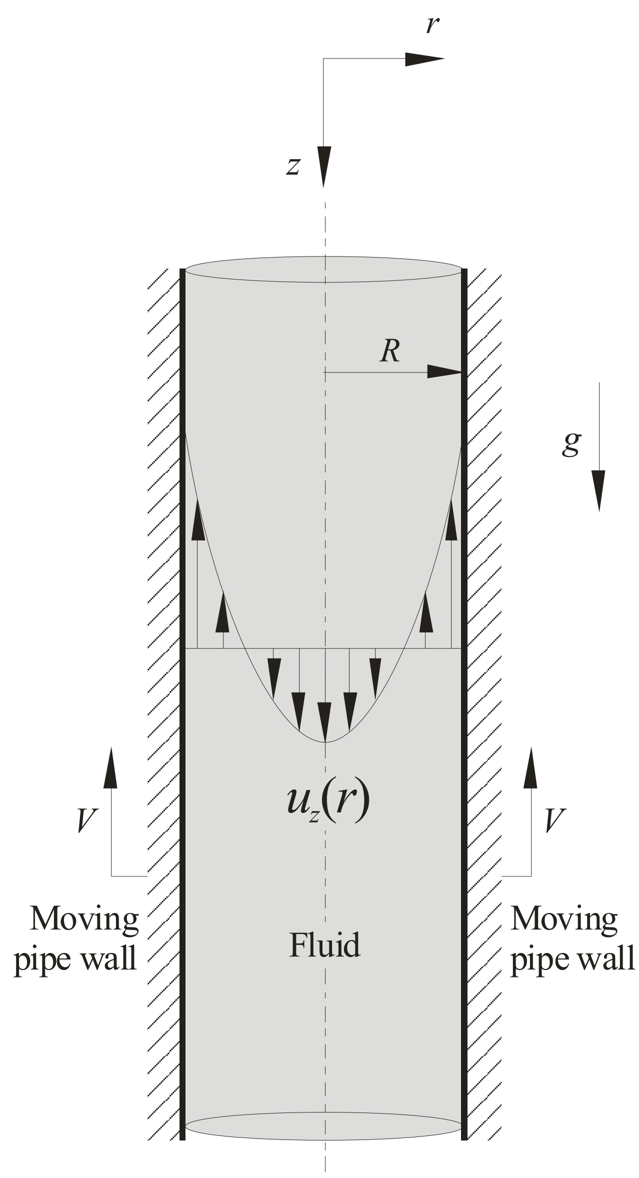


Figure 2

Problem 5

A thin sheet of smooth flat plywood lies flat on a rooftop, as shown in Figure 3 below. The plywood has dimensions 3 m × 2 m. The weight of the plywood is 90 N. The coefficient of static friction between the roof and the plywood is $\mu_S = 0.12$. Wind at standard atmospheric conditions (use the air properties given on the last page of the exam) blows parallel to the roof and plywood in the direction indicated in Figure 3. A boundary layer forms on the rooftop with its origin ($x = 0$) at the rooftop edge 2 m ahead of the plywood.

a. Referring to Figure 3, what wind velocity U will generate enough friction to dislodge the plywood from the roof? The thickness of the plywood can be neglected. In arriving at your answer, make use of one of the correlations provided on the last page of the exam. Be sure to justify your choice of correlation.

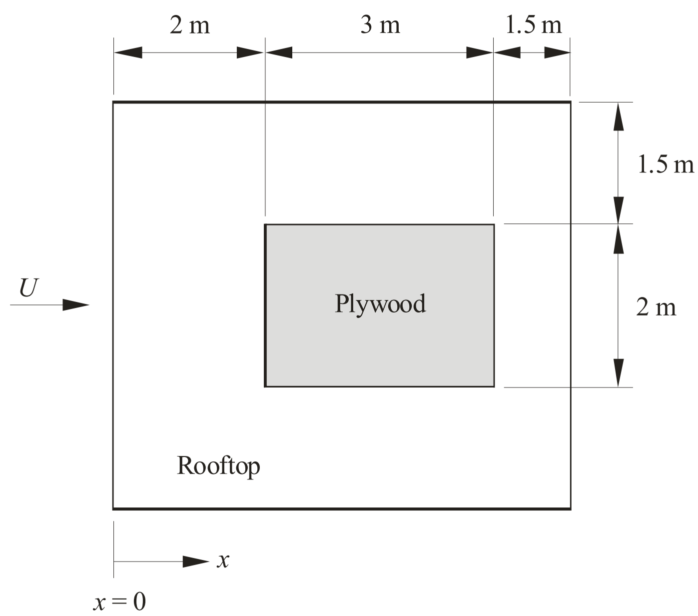


Figure 3

The same piece of plywood is now erected as a sign facing normal to the wind, as shown in Figure 4 below. The wind speed is 10 m/s. The plywood is mounted on a smooth cylindrical pole that is 0.15 m in diameter and 3 m high.

b. Calculate the bending moment at the base of the pole caused by the wind force. Clearly indicate any assumptions. Some useful information is found in Figure 5 and Table 1.

c. If the surface of the cylindrical pole was roughened, with an average roughness height of ε = 1 mm, determine the reduction in the bending moment. Clearly indicate any assumptions. From an engineering standpoint, is it worth roughening the surface? With the aid of a clearly labeled sketch, provide a physical explanation for the reduction in bending moment.

d. After several years of exposure to moderate wind speeds, the bolts holding the sign experience failure caused by fatigue. With the aid of a clearly labeled sketch, explain the origins of the fatigue failure from a fluid mechanics perspective.

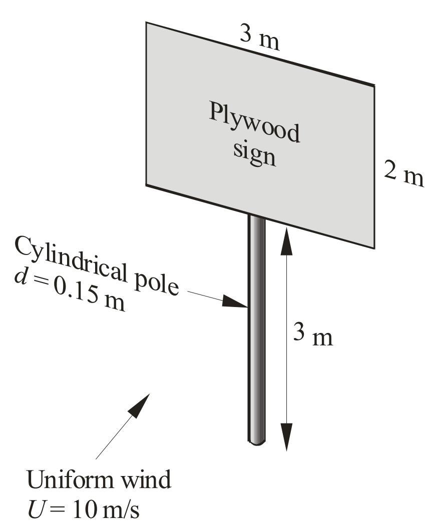


Figure 4

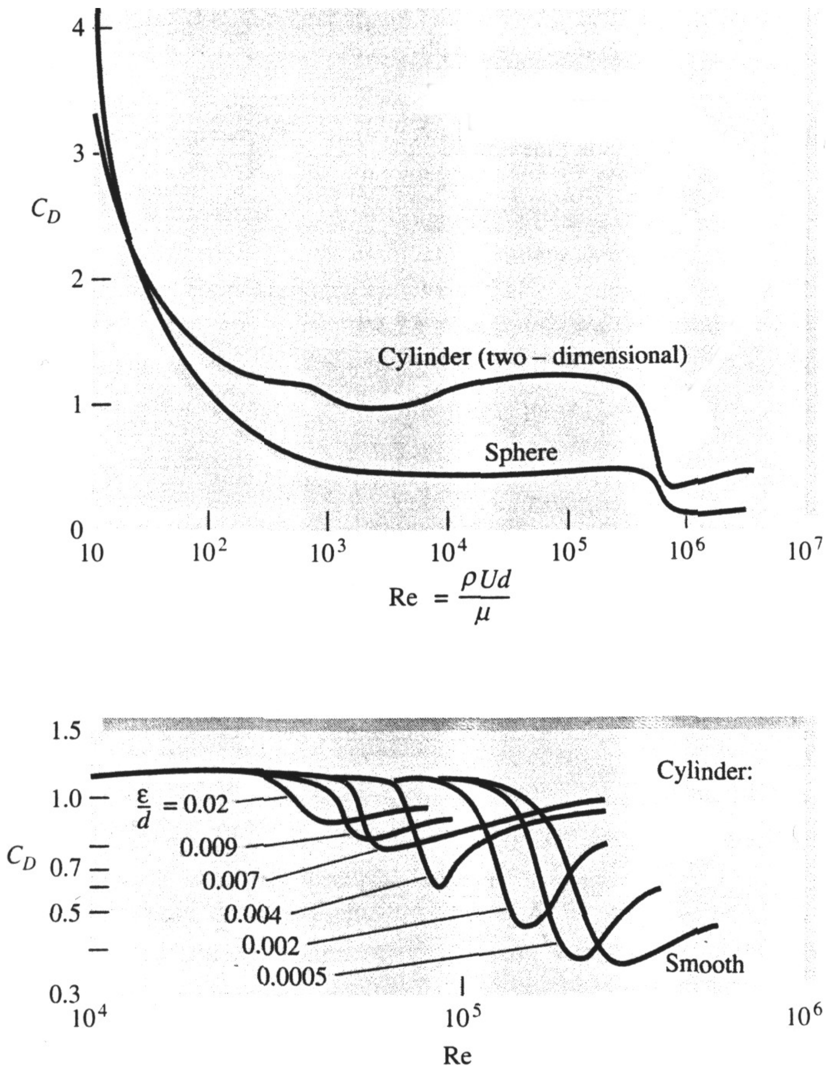
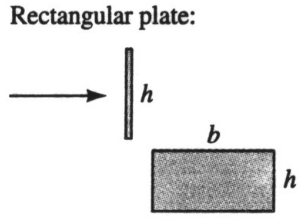


Figure 5

Table 1
(data are independent of Re)

Body	Ratio	C_D based on frontal area
Rectangular plate:		
	b/h 1	1.18
	5	1.2
	10	1.3
	20	1.5
	∞	2.0

Reference: F. M. White, 1999, *Fluid Mechanics*, 4th Edition, Boston: McGraw-Hill.

Problem 6

a. What is meant by the term "steady flow"?

b. What is the "Principle of Dimensional Homogeneity"?

c. Without writing any equations, provide a physical description of the boundary layer displacement thickness, δ^*.

d. With the aid of a clearly labeled sketch, explain how a laminar boundary layer undergoes transition to a turbulent boundary layer.

e. What is the universally accepted value of the critical Reynolds number for the transition of *any* laminar flow to turbulent flow?

f. What is meant by the "separation" of a boundary layer? Explain how pressure gradient influences separation. Give two examples of engineering flows that illustrate the consequences of boundary layer separation.

g. Why does a turbulent boundary layer have a higher skin-friction drag than a laminar boundary layer?

SOME USEFUL EQUATIONS AND INFORMATION

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \vec{\mathbf{V}}\right) = 0$$

$$\rho \frac{\partial \vec{\mathbf{V}}}{\partial t} + \rho\left(\vec{\mathbf{V}} \cdot \nabla\right)\vec{\mathbf{V}} = -\nabla P + \rho \vec{\mathbf{g}} + \mu \nabla^2 \vec{\mathbf{V}}$$

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} \qquad \tau_{yy} = 2\mu \frac{\partial v}{\partial y} \qquad \tau_{zz} = 2\mu \frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \quad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\rho c \frac{\partial T}{\partial t} + \rho c\left(\vec{\mathbf{V}} \cdot \nabla\right)T = k\nabla^2 T + \Phi$$

where $\Phi = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right]$

Cylindrical Coordinates, Incompressible Newtonian Fluid Flow

Gradient: $\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{e}}_{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathbf{e}}_{\boldsymbol{\theta}} + \frac{\partial f}{\partial z}\hat{\mathbf{e}}_{\mathbf{z}}$

Continuity: $\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta) + \frac{\partial}{\partial z}(u_z) = 0$

Convective time derivative: $\vec{\mathbf{V}} \cdot \nabla = u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}$

Laplacian operator: $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

r-momentum: $\frac{\partial u_r}{\partial t} + \left(\vec{\mathbf{V}} \cdot \nabla\right)u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + g_r + \frac{\mu}{\rho}\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)$

θ-momentum: $\frac{\partial u_\theta}{\partial t} + \left(\vec{\mathbf{V}} \cdot \nabla\right)u_\theta + \frac{u_\theta u_r}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + g_\theta + \frac{\mu}{\rho}\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)$

z-momentum: $\frac{\partial u_z}{\partial t} + \left(\vec{\mathbf{V}} \cdot \nabla\right)u_z = -\frac{1}{\rho}\frac{\partial P}{\partial z} + g_z + \frac{\mu}{\rho}\left(\nabla^2 u_z\right)$

Vorticity components: $\omega_r = \frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \qquad \omega_\theta = \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \qquad \omega_z = \frac{1}{r}\left(\frac{\partial (r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right)$

Flat-Plate Boundary Layer Correlations

Blasius' Solution: $C_f(x) = \frac{0.664}{\mathrm{Re}_x^{1/2}}$ 1/7th Power Law: $C_f(x) = \frac{0.027}{\mathrm{Re}_x^{1/7}}$

Properties of Air at Standard Conditions (p_{atm} = 101 kPa, T = 20°C)

ρ = 1.2 kg/m^3, μ = 1.8×10^{-5} Ns/m^2

Properties of Water at 20°C

ρ = 998 kg/m^3, μ = 1.01×10^{-3} Ns/m^2