University of Saskatchewan
Department of Mechanical Engineering
ME 352 Engineering Analysis (III)
Mid-term Examination
February 13, 2008

Name:	ID:

Instructions:

1. This is a two-hour test. You may use the following aids only: a formula sheet and a calculator.
2. Attempt all questions; their values are given in the table below.
3. Answer each question in the space provided; use the other side of the page if necessary.

Question	Mark Earned	Full Mark
#1 (Analysis)	6	6
#2 (Analysis)	4	6
#3 (Analysis)	7	8
#4 (Analysis)	7	8
#5 (Design)	3	12
Total	27	40

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<u>Problem 1: (6 Marks)</u> A system can be modeled by the following equation:

$$y(t) = 0.3r(t) + 10,$$

where $r(t)$ is the input signal to the system, $y(t)$ is the output response of the system.

<u>Is the above system linear or not? Justify your answer.</u>

1) Choose an input. let $r(t) = 5$.

2) Calculate corresponding out put $y(5) = .3(5) + 10 = 11.5$

3) chose another input let $r(t) = 10$.

4) Calalute corresponding output $y(10) = .3(10) + 10 = 13$

5) Choose single input $(x_1 + x_2) = 5 + 10 = 15$

6) Calalate corresponding out put of $(x_1 + x_2) = y(15) = .3(15) + 10$
$= 14.5$

7) check superposition Does $y = y_1 + y_2$
$14.5 \neq 11.5 + 13$.

the above equation is not linear as it fails the superposition check.

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Problem 2: (6 Marks) A system can be modeled by the following differential equation with some initial conditions:

$$\ddot{y}(t) + \dot{y}(t) = e^{2t}, \quad y(0) = 2, \quad \dot{y}(0) = 0.$$

Using the Laplace Transform method, find the solution of the above equation, i.e., the response of the system - $y(t)$.

$\ddot{y}(t) + \dot{y}(t) = e^{2t}$

$\left[\mathcal{L}\{\ddot{y}(t)\} + \mathcal{L}\{\dot{y}(t)\} = \mathcal{L}\{e^{2t}\}\right]$ $u(t)$

$= s^2Y(s) - Sy(0^-) - \dot{y}(0^-) + SY(s) - y(0) = \frac{1}{s-2}$

$= S^2Y(s) - S(2) + SY(s) - 2 = \frac{1}{s-2} + 2$

$(2s+2)(s-2)$
$2s^2 + 2s + 2s - 4$
$2s^2 + 4s - 4$

$Y(s)[s^2 - s] = \frac{1}{s-2} + 2 + 2s$

$Y(s)[s^2 - s] = \frac{1 + 2s^2 + 4s + 4}{s-2}$ $\Rightarrow Y(s) = \frac{2s^2 + 4s + 5}{(s^2 - s)(s-2)}$

$s^3 - 2s^2 - s^2 + 2s$
$-3s^2$

$2s^2 - 2s - 3$

$Y(s) = \frac{2s^2 + 4s + 5}{s^3 - 3s^2 + 2s}$ $\Rightarrow$ $\frac{}{s(s^2 - 3s + 2)}$

$s(s-2)(s+1)$

$Y(s) = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+1}$ $=$ $A(s-2)(s-1) + B(s)(s-1) + C(s)(s-2) =$ Numerator

$s = 2$; $B(2)(1) = 21$ $\quad B = \frac{21}{2}$

$s = 1$; $C(1)(-1) = 11$ $\quad C = -11$

$s = 0$; $A(-2)(-1) = 5$ $\quad A = \frac{5}{2}$

$Y(s) = \frac{5/2}{s} + \frac{\frac{21}{2}}{s-2} - \frac{11}{s-1}$

$\boxed{y(t) = \frac{5}{2} + \frac{21}{2}e^{2t} - 11e^{t} \quad t \geq 0}$

$= \frac{3}{2} + \frac{1}{3}e^{-t} + \frac{1}{6}e^{2t}$.

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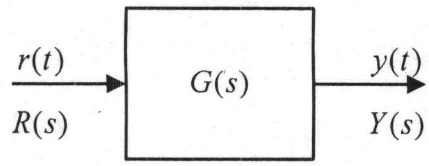
Problem 3: (8 Marks) An open loop control system is shown in the following figure. $y(t)$ is the output response, $r(t)=\delta(t)$ [$\delta(t)$ is the impulse signal], $G(s)=\dfrac{1}{s^4+4}$.

Determine the output response — $y(t)$.

$Y(s)=G(s)R(s)$



$r(t)=\delta(t)$

$R(s)=1$

$$Y(s)=\frac{1}{s^4+4}(1) \Rightarrow \frac{1}{s^4+(2s)^2+2^2-(2s)^2}=\frac{1}{(s^2+2)^2-(2s)^2}$$

using $A^2-B^2=(A+B)(A-B)$

$$\frac{1}{(s^2+2s+2)(s^2-2s+2)} \quad (1)\ \frac{As+B}{s^2+2s+2} + (2)\ \frac{Cs+D}{s^2-2s+2}$$

$s^2+2s+2+1-1$

s^2+2s+1

$(s+1)(s+1)-1$

$(s+1)(s-1)$

$A=\frac{1}{8}$ $B=\frac{1}{4}$ $C=-\frac{1}{8}$ $D=\frac{1}{4}$

$$\frac{\frac{1}{8}s+\frac{1}{4}}{s^2+2s+2}\ (1) + \frac{-\frac{1}{8}s+\frac{1}{4}}{s^2-2s+2}\ (2)$$

s^2+2s+2

$As^3-2As^2+2As+\cancel{Bs^2}$
$-2Bs+2B$
$+Cs^3+2Cs^2+2Cs+Ds^2+2Ds+2D$

$As^3+Cs^3=0$

$-2As^2+Bs^2+2Cs^2+Ds^2=0$

$+2As-2Bs+2Cs+2Ds=0$

$2B+2D=1$

Using the formula from class given $\frac{As+B}{s^2+Cs+D}$

$a=C/2$

$\omega=\sqrt{D-\left(\frac{C}{2}\right)^2}$

$\dfrac{B-\frac{AC}{2}}{\omega}$

(1) $a=1$

$\omega=\sqrt{2-(1)^2}=1$

$\dfrac{\frac{1}{4}-\frac{(\frac{1}{8})(2)}{2}}{1}=\frac{1}{8}$

$\frac{1}{8}e^{-t}\cos t+\frac{1}{8}e^{-t}\sin t$

(2) $a=-1$

$w=\sqrt{2-1}$

$w=1$

$\frac{1}{4}-\frac{(-\frac{1}{8})(2)}{2}$

$=\frac{3}{8} \rightarrow \frac{1}{8}$

$-\frac{1}{8}e^{t}\cos t+\frac{3}{8}e^{t}\sin t$

$$Y(t)=\frac{1}{8}e^{-t}\cos t+\frac{1}{8}e^{-t}\sin t-\frac{1}{8}e^{t}\cos t+\frac{3}{8}e^{t}\sin t$$

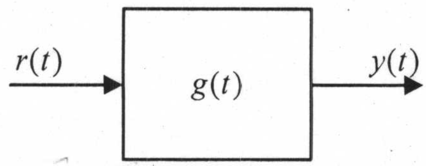
Problem 4: (8 Marks) An open loop control system is shown in the following figure. In time-domain, the output response can be represented by

$$y(t) = g(t) * r(t),$$

where $y(t)$ is the output response, $r(t) = u(t)$ [$u(t)$ is the unit step signal], the transfer function of the plant or the Laplace transform of $g(t)$ is $G(s) = \dfrac{4s-10}{s^2+6s+10}$, and *the star* operator * is the convolution. **Calculate the output response** — $y(t)$.

$r(t) = u(t)$

$R(s) = \frac{1}{s}.$



$Y(s) = G(s)\,R(s).$

$Y(s) = \frac{4s-10}{s^2+6s+10} \cdot \frac{1}{s}.$

$Y(s) = \frac{4s-10}{s(s^2+6s+10)} \rightarrow \frac{A}{s} + \frac{Bs+C}{s^2+6s+10}.$

$As^2 + 6As + A10 + Bs^2 + Cs = 4s - 10$

$A + B = 0$

$6A + 1C = 4.$

$10A = -10$

$A = -1$

$B = 1$

$6(-1) + C = 4$

$-6 + C = 4$

$C = 10.$

$\frac{-1}{s} + \frac{1s + 10}{s^2+6s+10}.$

$a = \frac{6}{2} = 3$

$w = \sqrt{10 - \left(\frac{6}{2}\right)^2}$

$= \sqrt{10 - 9}.$

$w = 1$

$B - \frac{AC}{2} = 10 - \frac{(1)(6)}{2}$

$= 7$

$$\boxed{y(t) = -1 + e^{-3t}\cos t + 7e^{-3t}\sin t \quad t \geq 0}$$

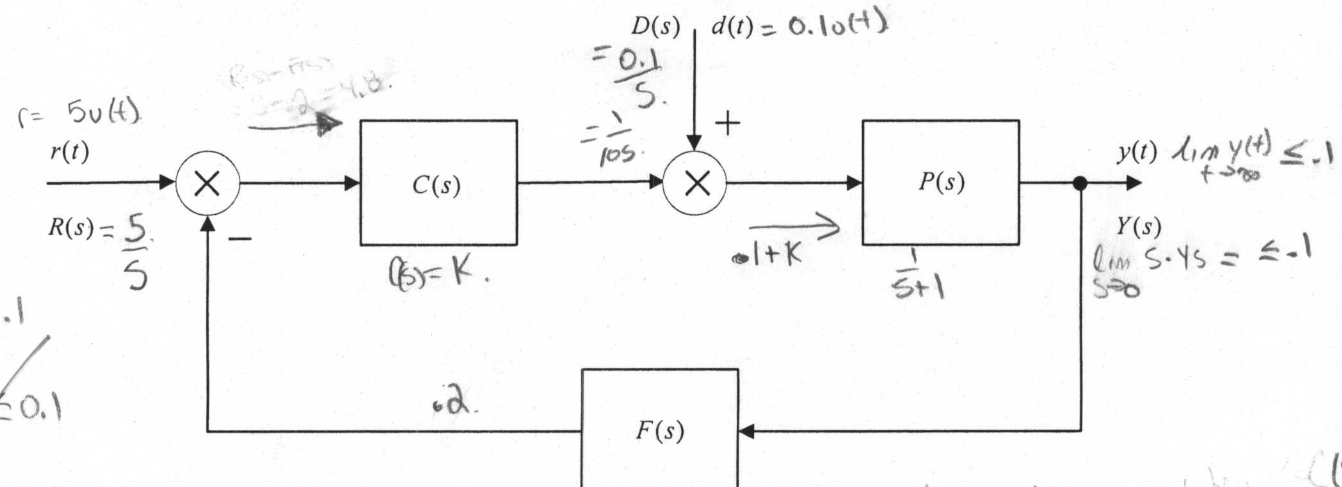
Problem 5: (12 Marks) In the following linear control system, $r(t)$ is the reference input signal, and $d(t)$ is the disturbance signal.

- If $d(t)=0,\ r(t)\neq 0$, then $Y(s)=\dfrac{P(s)C(s)}{1+P(s)C(s)F(s)}R(s)$; ①
- If $r(t)=0,\ d(t)\neq 0$, then $Y(s)=\dfrac{P(s)}{1+P(s)C(s)F(s)}D(s)$. ②

Suppose we have the following information about the system:

- Reference input signal: $r(t)=5u(t)$;
- Disturbance signal: $d(t)=0.1u(t)$; $\mathcal{L}(d(t)) = .1 \quad t\geq 0.$
- Transfer function of the plant: $P(s)=\dfrac{1}{s+1}$;
- Transfer function of the sensor: $F(s)=0.2$;
- Transfer function of the controller: $C(s)=K$ (K is a constant to be designed).

Design the controller: If the control objective requires that the steady state value of the output response $\lim_{t\to\infty} y(t)\leq 0.1$ (assume that the stead state value does exist). Please design the controller to satisfy the control objective.



$\lim_{t\to\infty} y(t)\leq 0.1$

$=\lim_{s\to 0} s\cdot Y(s)\leq 0.1$

① = ②

$$Y(s)=\frac{\left(\frac{1}{s+1}\right)(K)}{1+\left(\frac{1}{s+1}\right)(K)(.2)}\left(\frac{5}{\cancel{s}}\right) = \frac{\left(\frac{1}{s+1}\right)}{1+\left(\frac{1}{s+1}\right)(K)(.2)}\frac{1}{10\cancel{s}}$$

$$=\frac{\frac{5K}{s+1}}{1+\frac{.2K}{s+1}} = \frac{\frac{1}{s+1}}{10+\frac{2K}{s+1}}$$

$$=\frac{\frac{5K}{s+1}}{\frac{s+1+.2K}{s+1}} = \frac{\frac{1}{s+1}}{\frac{10s+10+2K}{s+1}}$$

$$\frac{5K}{s+1+.2K} = \frac{1}{10s+10+2K}.$$

$$5K\,\frac{10s+10+2K}{s+1+.2K} = \frac{1}{5K}$$

$10.$

$50K = 1.$

$$\boxed{C(s)=\frac{1}{50}.}$$

See Jan a(?)'s Notes.