# University of Saskatchewan
# Department of Mechanical Engineering
# ME 352 Engineering Analysis (III)
# Mid-term Examination
# February 13, 2008

| Name: | ID: |
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Instructions:

1. This is a two-hour test. You may use the following aids only: a formula sheet and a calculator.
2. Attempt all questions; their values are given in the table below.
3. Answer each question in the space provided; use the other side of the page if necessary.

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| Question | Mark Earned | Full Mark |
|---|---|---|
| #1 (Analysis) | 6 | 6 |
| #2 (Analysis) | 4 | 6 |
| #3 (Analysis) | 7 | 8 |
| #4 (Analysis) | 7 | 8 |
| #5 (Design) | 3 | 12 |
| Total | 27 | 40 |

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**Problem 1: (6 Marks)** A system can be modeled by the following equation:

$$y(t) = 0.3r(t) + 10,$$

where $r(t)$ is the input signal to the system, $y(t)$ is the output response of the system.

**Is the above system linear or not? Justify your answer.**

1) Choose an input. Let $r(t) = 5$.

2) Calculate corresponding output $y(5) = .3(5) + 10 = 11.5$

3) Choose another input Let $r(t) = 10$.

4) Calculate corresponding output $y(10) = .3(10) + 10 = 13$

5) Choose single input $(x_1 + x_2) = 5 + 10 = 15$

6) Calculate corresponding output of $(x_1 + x_2)$ = $y(15) = .3(15) + 10$
$= 14.5$

7) check superposition Does $y = y_1 + y_2$
$14.5 \neq 11.5 + 13$.

The above equation is not linear as it fails the superposition check.

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**Problem 2: (6 Marks)** A system can be modeled by the following differential equation with some initial conditions:

$$\ddot{y}(t) + \dot{y}(t) = e^{2t}, \quad y(0) = 2, \quad \dot{y}(0) = 0.$$

Using the Laplace Transform method, find the solution of the above equation, i.e., the response of the system - $y(t)$.

$\ddot{y}(t) + \dot{y}(t) = e^{2t}$

$\left[\mathcal{L}\{\ddot{y}(t)\} + \mathcal{L}\{\dot{y}(t)\} = \mathcal{L}\{e^{2t}\}\right]$ U(t)

$= s^2 Y(s) - sy(0^-) - \dot{y}(0^-) + sY(s) - y(0) = \frac{1}{s-2}$

$= s^2 Y(s) - s(2) + sY(s) - 2 = \frac{1}{s-2} + 2$

$Y(s)[s^2 - s] = \frac{1}{s-2} + 2 + 2s$

$(2s+2)(s-2)$
$2s^2 + 2s + 2s - 4$
$2s^2 + 4s - 4$

$Y(s)[s^2 - s] = \frac{1 + 2s^2 + 4s + 4}{s-2} \Rightarrow Y(s) = \frac{2s^2 + 4s + 5}{(s^2 - s)(s-2)}$

$2s^2 - 2s - 3$

$s^3 - 2s^2 - s^2 + 2s$
$-3s^2$

$Y(s) = \frac{2s^2 + 4s + 5}{s^3 - 3s^2 + 2s} \Rightarrow \frac{}{s(s^2 - 3s + 2)}$

$s(s-2)(s+1)$

$Y(s) = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+1}$ $= A(s-2)(s-1) + B(s)(s-1) + C(s)(s-2) =$ Numerator

$s = 2$ ; $B(2)(1) = 21$ $\quad B = \frac{21}{2}$

$s = 1$ ; $C(1)(-1) = 11$ $\quad C = -11$

$s = 0$ ; $A(-2)(-1) = 5$ $\quad A = \frac{5}{2}$

$Y(s) = \frac{5/2}{s} + \frac{21/2}{s-2} - \frac{11}{s-1}$

$= \frac{3}{2} + \frac{1}{3}e^{-t} + \frac{1}{6}e^{2t}$

$y(t) = \frac{5}{2} + \frac{21}{2}e^{2t} - 11e^{t} \quad t \geq 0$