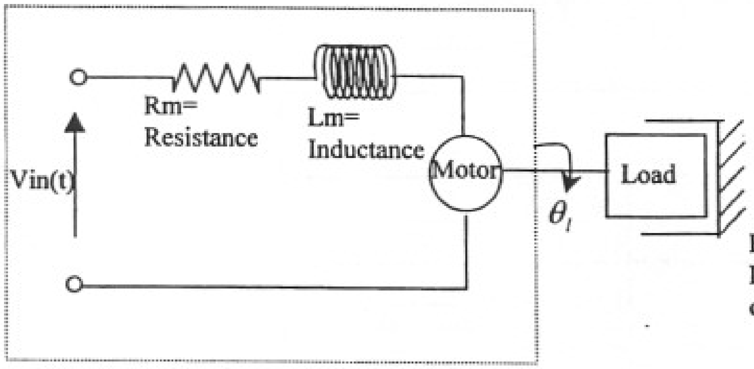
ME 431/AE411 Control Systems I
Final
December 1999

Time: 3 hours
Instructor: S. Habibi

Instructions: **1- Answer any 4 questions**
2- Calculators are allowed
3- Students may bring up to 2 pages of letter-size notes

(25) Q1: An electromechanical system is shown:



B = Damping coefficient

Constant Field DC Motor With
θ_l=angular position
Kt=motor torque constant
Ke= constant relating back emf to motor angular velocity.
Tm= motor torque
I= Current
(some formulas:
Tm=Kt * I;
Back emf= Ke * angular velocity)
Jl=Motor/Load Inertia

Figure 1

a) Write the differential equations of the electromechanical system in Figure 1 and find their Laplace transform.
b) Draw a block diagram representation of the system in Figure 1.
c) What are the type and order of the system in Figure 1?
d) For: $Jl = 0.002 Kg - m^2$, Rm=0.5Ω, Lm=0.004 H, $Ke = 0.6 V/rad/s$, $B = 0.02 Nm/rad/s$, and $Kt = 1 Nm/A$, reduce the order of this model to contain only its dominant dynamic characteristics. Justify your answer and, write down the transfer function of your reduced order model.
e) Obtain the transfer function $\dfrac{Y(s)}{U_1(s)}$ of the system represented by the following block diagram (Figure 2).

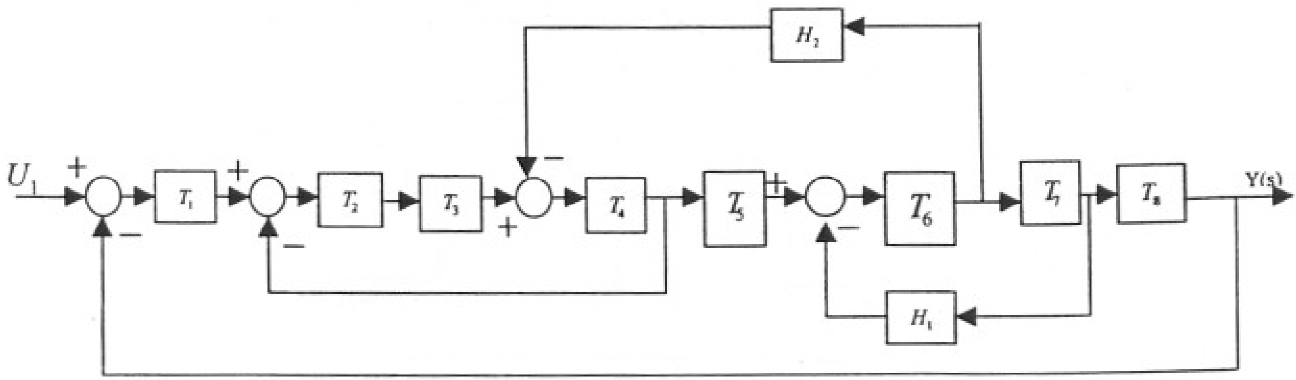
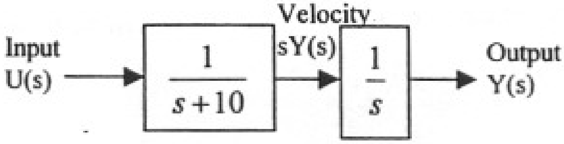


Figure 2

(25) Q2: Consider the open loop System represented by the following block diagram:



a) Use Bode diagrams to determine the gain and phase margins of the above system.
b) Design a proportional controller with unity feedback and, determine the maximum possible proportional gain that would provide a gain margin of <u>at least</u> 8 dB and a phase margin of <u>at least</u> 45 degrees.
c) For U(s)= $\frac{1}{s}$, what is the steady state error of the closed loop system obtained from b).
d) Determine the natural frequency and the damping ratio of the closed loop system obtained from b).
e) How can velocity feedback be used for changing the damping ratio of your system. Determine the velocity feedback gain K_v that is needed to obtain critical damping.

(25) Q3: Use the Routh Hurowitz criterion to determine the number of roots in the left half-plane, the right half plane, and on the imaginary axis for the following characteristic equations:

a) $s^4 + 2s^3 + 3s^2 + 4s + 5$
b) $s^3 + 2s^2 + s + 2$
c) $2s^3 + s^2 - 3s + 10$

Determine the range of K_p and K_i for stability of the following characteristic equations:

d) $s^3 + 3s^2 + 2s + K_p$
e) $s^4 + 4s^3 + 5s^2 + (2 + 2K_p)s + 2K_i$

(25) Q4: Plot the Bode gain and phase diagrams for the following transfer functions:

a) $\dfrac{\left(\frac{s}{1000}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$

b) $\dfrac{10s^2}{(s+1)^2}$

c) $\dfrac{100}{(s^2+18s+100)}$

d) $\dfrac{10(-s+100)}{(s+10)(s+1000)}$

e) $\dfrac{e^{-10s}}{(s+10)}$

(25) Q5:

a) Draw the block diagram of a system with unity feedback given the following open loop transfer function:

$$G(s) = \frac{K(s+1)}{s(s-1)}$$

b) Draw the Nyquist Diagram of the system in a). Your answer should clearly show your derivations and the Nyquist contour that you have used.

c) Apply the Nyquist criterion to determine the range of values of K for which the system is:

- stable, and
- unstable.