ME 431/AE411/ME862 Control Systems I
Final Examination
December 2000

Time: 3 hours
Instructor: S. Habibi

Instructions: **1- <u>Answer any 4 questions</u>**
2- Calculators are allowed
3- Students may bring up to 2 pages of letter-size notes

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(25) Q1: An electromechanical system is shown:

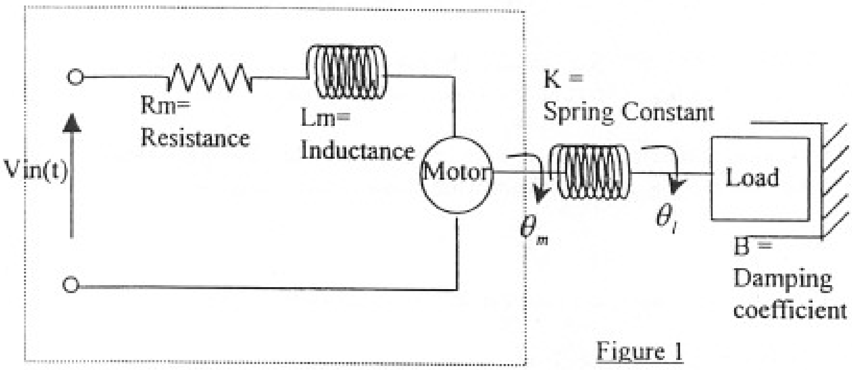


Figure 1

Constant Field DC Motor With
θ_m =motor angular position
θ_l =load angular position
Kt=motor torque constant
Ke= constant relating back emf to motor angular velocity.
Tm= motor torque
I= Current
(some formulas:
Tm=Kt * I;
Back emf= Ke * angular velocity)
Jm=Motor inertia
Jl=Load Inertia

a) Write the differential equations of the electromechanical system in Figure 1 and find their Laplace transform.
b) Draw a block diagram representation of the system in Figure 1.
c) For a step input of magnitude 2, determine the steady state error of a system with the following transfer function:

$$\frac{8}{s^2+4s+16}$$

d) Determine the damping ratio and natural frequency of the transfer function given in c).

(25) Q2: Indicate the break frequencies of each of the following transfer functions and, plot their Bode gain and phase diagrams:

a) $$\frac{\left(\frac{s}{10}+1\right)}{\left(\frac{s}{100}+1\right)}$$

b) $$\frac{1000}{(s+100)^2}$$

c) $$\frac{100s^2}{(s+1)^3}$$

d) Consider the transfer function $G(s)=s-10$.
   i) Derive the gain and phase equations of this transfer function.
   ii) Using these equations, calculate the gain and phase of $G(s)=s-10$ at the following frequencies: 0.1, 1, 10, 100, and 1000 rad/s.
   iii) Draw the bode gain and phase plots of $G(s)=s-10$.
   iv) Obtain the approximated gain and phase of $G(s)=s-10$ from the Bode plots at the following frequencies: 0.1, 1, 10, 100, and 1000 rad/s.

(25) Q3: Use the Routh Hurowitz criterion to determine the number of roots in the left half-plane, the right half plane, and on the imaginary axis for the following characteristic equations:

a) $s^3 + 2s^2 + 3s + 2$

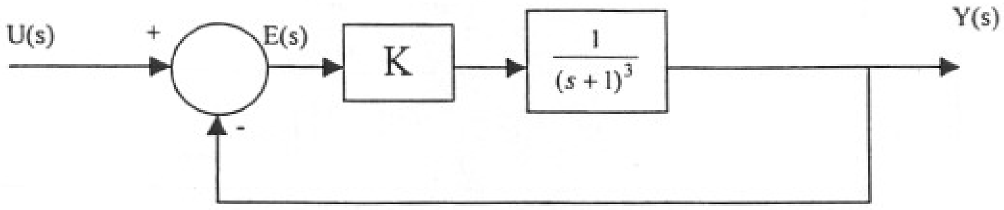
b) $s^4 + s^3 + 5s^2 + 5s + 2$

c) $s^4 + s^2 + 2$

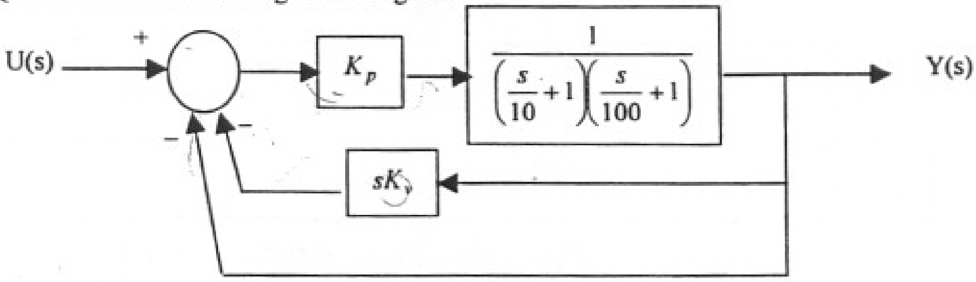
d) Determine the range of K_p for stability of a system with the characteristic polynomial:

$$s^3 + 3s^2 + 3s + (1 + 5K_p)$$

e) Determine the characteristic polynomial that must be used for determining the stability of the following closed loop system: (your answer should ONLY provide the polynomial)



(25) Q4: Consider the following block diagram:



a) For $K_v = 0$ and by using Bode gain and phase diagrams, design a proportional controller with the highest possible proportional gain K_p that would satisfy the following relative stability requirements:
   - a phase margin of 45 degrees, and
   - a gain margin greater than 8 dB.

b) Determine the transfer function, $\dfrac{Y(s)}{U(s)}$.

c) Adjust the velocity feedback gain K_v such that critical damping is obtained.

d) How would you modify your system such that, given a step input, a steady state error of zero is obtained.

(25) Q5: a) Draw the block diagram of a system with unity feedback given the following open loop transfer function:

$$G(s) = \frac{1}{(2s+1)(s^2+s+1)}$$

b) Derive clearly the equations relating the gain and phase of G(s) to a sinusoidal input signal with excitation frequency ω.

c) Calculate the gain of the transfer function when its phase angles are 0, -90 and –180 degrees respectively. Your answer should clearly indicate how this calculation is performed by using the equations derived in b).

d) Draw the Nyquist Diagram and the Nyquist Path of the system in a). Apply the Nyquist criterion for its stability analysis.

e) Find the gain margin from the Nyquist Diagram.