ME 431/AE411 Control Systems I
Midterm
October 2000

Time: 2 hours
Instructor: S. Habibi
Instructions: 1- Answer all 4 questions
2- Calculators are allowed
3- Students may bring up to 2 pages of letter-size notes

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(30) Q1: Consider the following block diagram:

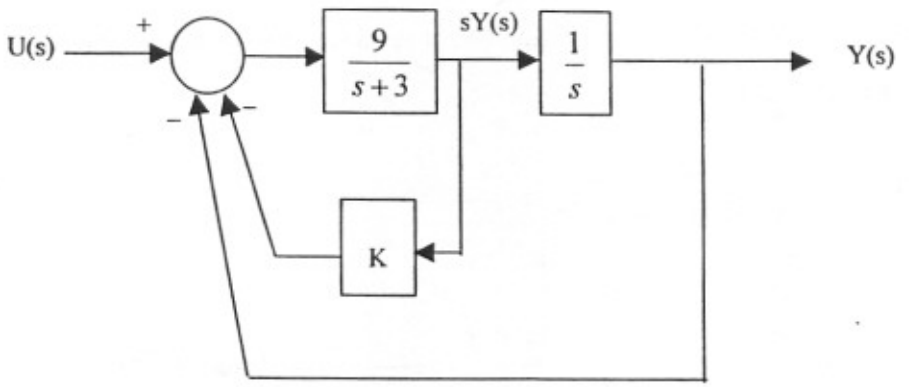


Figure 1

(Some formulas: $\tan\alpha=\frac{\sqrt{1-\zeta^2}}{\zeta}$, poles for a standard second order system are obtained at $s=-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$, time response of a second order system given a unity step input is $y(t)=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\sqrt{1-\zeta^2}\,\omega_n t+\alpha)$).

a) Determine the transfer function, $\frac{Y(s)}{U(s)}$ in the simplest form i.e obtain *a, b* and *c* in the simplified form of $\frac{a}{s^2+bs+c}$.

b) Find the damping factor, ζ, and natural frequency ω_n for K=0.

c) For K=0, what is the steady state value of the output when the system is given a step input of magnitude 5? What is the settling time?

d) Plot the poles of the transfer function for K=0 in a complex plane (Imaginary Vs Real) and show the angle α.

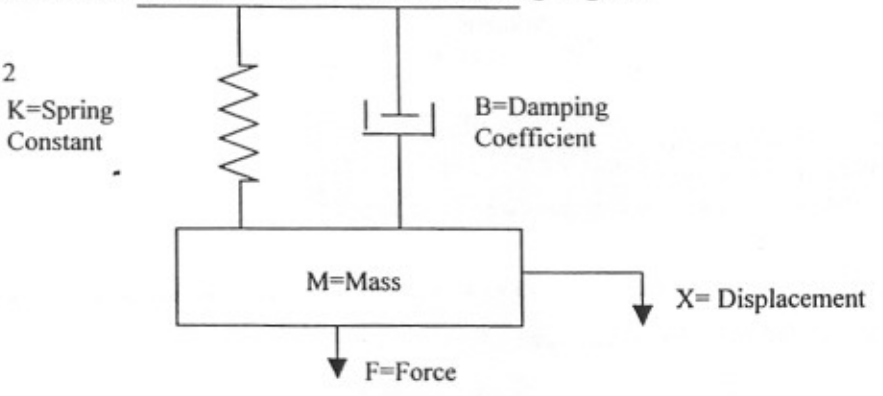
e) In a typical second order system, what would be the value of the damping factor, ζ, that would give critical damping. What is the value of K under this condition for the system in Figure 1?

(20) Q2: Calculate the gain and phase (in degrees) of the transfer function $T(s)$ at the following frequencies (rad/s): 0.01, 0.1, 0.5,1, 5, 10, 15, 20, 50, 100,1000.

$$T(s)=\frac{(s+10)}{(s+1)}$$

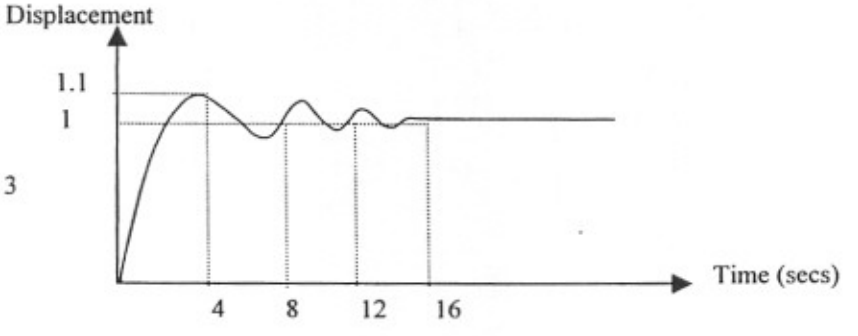
(30) Q3: A mechanical system is shown in the following diagram:

Figure 2



The time response of this system to a step input of magnitude 1 Newton is as follows:

Figure 3



a) Write the differential equations of the system in Figure 2.
b) Find the transfer function of this system.
c) Find the value of the damping coefficient ς and the natural frequency ω_n.
d) Find the value of Mass (M), Spring Constant (K) and Damping Coefficient (B).
e) Indicate the regions of Steady State and Transient Response of Figure 3.
f) If the amplitude of the step input is increased to 50 N, calculate the steady state value of the response.

(20) Q4: a) What is the meaning of the term *manipulated variable* in a control system.

b) Use the Mason's gain formula and the block diagram simplification method to obtain the transfer function $\frac{Y(s)}{U_1(s)}$ of the system represented by the following block diagram (Figure 4).

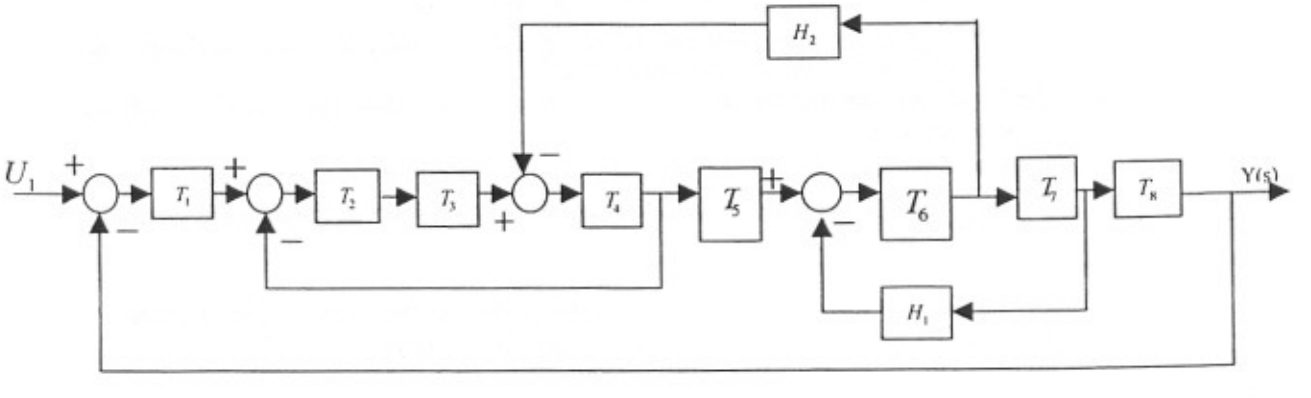


Figure 4