UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Closed-book examination — December 2005
3 double-sided sheets of formulas are allowed — W. Szyszkowski
Answer **three** questions only

Q1. You are expected to present detailed derivations and/or explanations as requested in the problems below.

a) The displacement of the 3-node bar element is approximated by $\tilde{u} = \alpha_0 + \alpha_1 \xi + \alpha_2 \xi^2$.
Show how to transform this parabolic approximation function with the parameters α_0, α_1, and α_2 into the form $\tilde{u}(\xi) = \sum_{i=1}^{3} N_i(\xi) u_i$, where $N_i(\xi)$ and u_i are the shape functions and the element's DOFs respectively. Obtain all $N_i(\xi)$.

b) The element's stiffness matrix can be defined as: $K_e = \int_{V_e} B^T D B dV$.

Briefly explain the meaning of matrices B and D.
Specify the size of matrix B (the number of rows and columns) for each of the 2-D elements listed:

- the triangular 3-nodes element to analyze plane stress problems,
- the triangular 3-nodes element to analyze axisymmetric stress problems,
- the quadrilateral 4-nodes element to analyze plane stress problems.

For which of the above elements matrix B is constant and why?
Also, specify the size of matrix K_e for the above elements.

c) The buckling problem in beams is governed by the differential equation:

$$\frac{d^2}{dx^2}\left(EI \frac{d^2 v}{dx^2}\right) - \underline{\frac{d}{dx}\left(n \frac{dv}{dx}\right)} = 0$$

Use Galerkin's method to show that the underlined term renders the geometrical stiffness matrix with the terms: $K_{ij}^g = \int_{L_e} n \frac{dN_i}{dx} \frac{dN_j}{dx} dx$

Use direct integration to calculate K_{11}^g if the element axial force is $n = n_e = \text{const}$, and $N_1 = (2 - 3\xi + \xi^3)/4$.
How many Gauss points would be necessary to obtain K_{11}^g exactly via the Gauss quadratures (instead of integrating directly)?

Q2. Apply *two* elements of equal length to analyze the beam shown (note that the slope and vertical displacement at node 3 are zero).

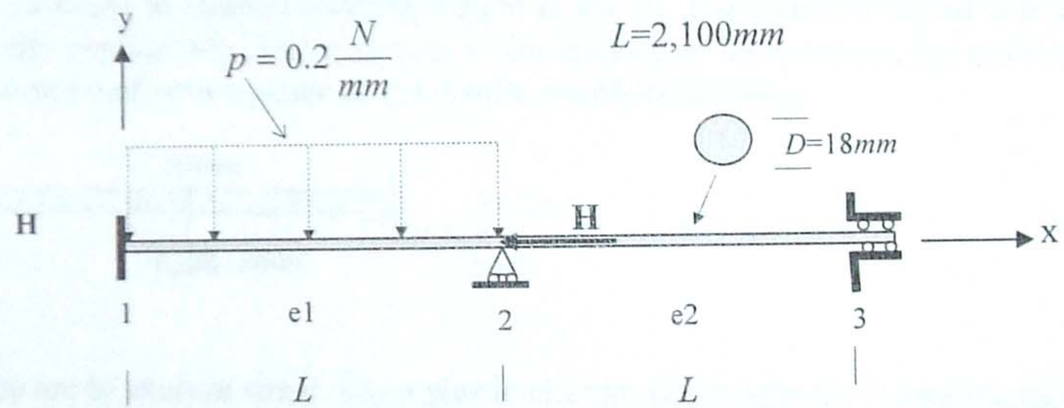
a) Determine the slope and the reaction at node 2 due to load *p*.
Sketch the deformed configuration of the beam.
For comparison, sketch the displacement plot that would be obtained from ANSYS if two *beam3* elements were used, and then if twenty *beam3* elements were used.

b) The nodal values of the bending moments and shear forces at nodes 1 and 3 are:

$$M_1^1 = 91{,}875Nmm\ ,\ V_1^1 = 236.25N \quad M_3^2 = 18{,}375Nmm\ ,\ V_3^2 = -26.25N\ .$$

Determine the nodal values of the bending moments and shear forces at node 2 and sketch the ***M*** and ***V*** diagrams for the whole beam (use the ANSYS plotting convection).
Check the equilibrium of node 2.
For this case, how accurate should be the diagrams obtained from two elements solution?
Use your diagrams to calculate the maximum bending stress,
and the maximum average shear stress.

c) Apply the geometrical stiffness matrix to solve the buckling problem if a horizontal force **H**=10,000*N* is acting at node 2 as shown.
Determine the safety factor for stability. Sketch the buckling mode.
How would the factor and the buckling mode be affected if the beam was meshed with 20 elements?



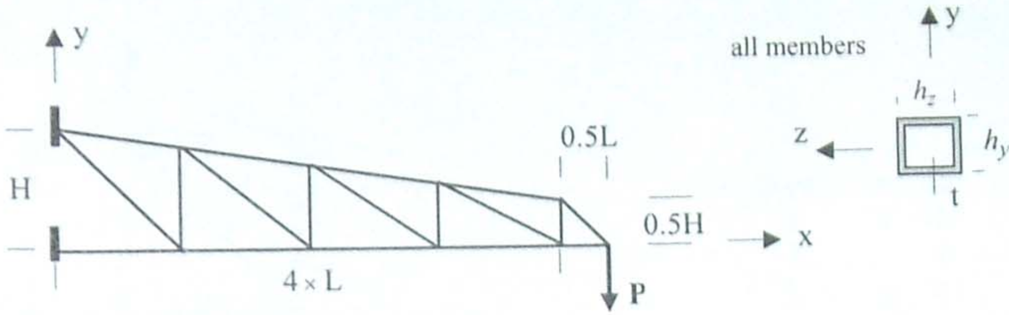
Given: E=200,000MPa, $I = \frac{\pi}{64} D^4 = 5153mm^4$

Q3. (a) The boom structure shown can be modeled by several different elements available in ANSYS. Using this example explain in detail pros & cons of applying the following:

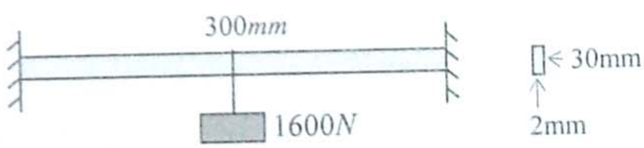
1) ***link1*** - the bar element in 2-D,
2) ***link8*** - the bar element in 3-D,
3) ***beam3*** - the beam element in 2-D,
4) ***beam4*** - the beam element in 3-D,
5) ***shell63*** - the 2-D element with in-plane and bending stiffness.

In particular, describe how the above element types would affect the calculated maximum displacements, maximum stresses, and the safety factor for stability.

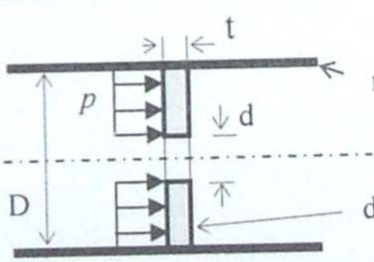
In *lab#1* each member of the boom was modeled by *one link* element, and in *lab#4* each member was modeled by *five beam* elements. What would happen if *five link* elements per member were used in lab #1, and *one beam* element per member was used in lab#4?



(b) Several meshing patterns and different types of 2-D elements were used in *lab#2* to analyze the structure to support a 1600*N* weight as shown. Some results varied in a wide range. Briefly explain why, and give your recommendation for modeling the problem so that accuracy and convergence of the results would be achieved.



(c) You are to analyze stresses in a plastic circular diaphragm fixed inside a rigid tube to control the flow of a fluid. The fluid pressure p=0.25*MPa* is acting as shown.



E=4*GPa*, $\nu = 0.4$
D=80*mm*, d=10*mm*
t=6*mm*

What element type would you use? What size of the elements would you recommend? Specify the keypoints and list their locations:

K_i	x_i	y_i
1	......	.....

Define the lines and describe how the load and boundary condition are applied.