UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Open-book examination
Answer **four** questions only

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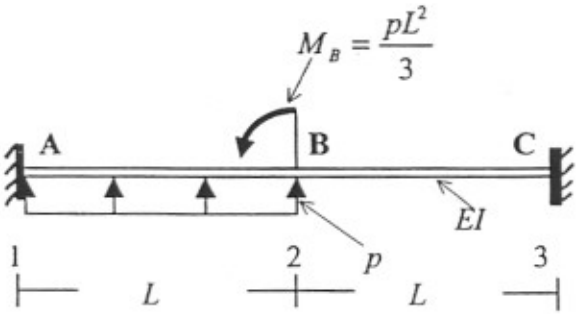
Q1. Use two elements to solve the beam fixed at **A** and **C** and loaded as shown.
Determine, in terms of EI, L and p:
a) The deflection and slope at **B** (sketch the shape of the deflected beam).
b) The reactions at **A** and **C** (check the equilibrium).
c) The bending moment and the shear force for the elements (plot the M and V diagrams).
d) The maximum deflection in the section **AB**.
e) The maximum deflection in the section **BC**.
How accurate are your results?



Q2. Briefly answer the following questions:

a) Why does the *quadratic* bar element need *three* nodes, while the beam element (which uses *cubic* polynomials) requires only *two* nodes?

b) Describe *two* advantages and *two disadvantage* (the most important, in your opinion) of the use of *linear* elements versus the *quadratic* ones.

c) A 2×2 Gauss integration scheme is typically used to obtain K_e for the linear quadrilateral elements. For what geometrical shape of the element the result of the Gauss integration will be exact and why? Would the 3×3 integration scheme be exact for an arbitrary shape of the element?

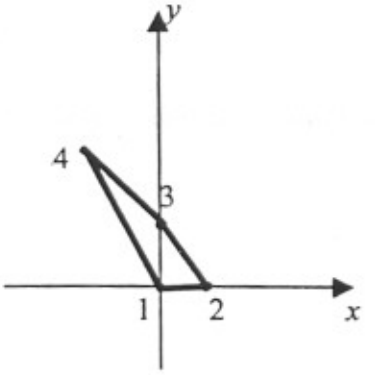
d) Why is the 3-node triangular element for plane problems referred to as the *CST*-element? If this element is to be applied to an axisymmetric problem, which of the following should be modified and why (assume identical nodal coordinates) to determine the new K_e:

- the shape functions **N**, - the Jacobian **J**, - the strain-displ. matrix **B**, - the constitutive law matrix **D**? Is the name *CST* justified for the axisymmetric applications?

e) The mass matrices for the bar and the beam elements were both derived in the form $\mathbf{M} = \int_{l_e} \mathbf{N}^T \rho A \mathbf{N} dx$. Explain why **M** is 2×2 matrix for the bar element, while **M** is 4×4 matrix for the beam element. What is the difference between the consistent and lumped **M**, and what are the consequences of using either of these matrices in the vibrations analysis?

f) By the direct integration, prove that the stiffness matrix term K_{13} for the beam element of a constant cross-sectional area is equal to $-12EI/l_e^3$.

Q3. Four nodes of the quadrilateral element shown are located as follows:



Node	x	y
1	0	0
2	1	0
3	0	1.1
4	-1	2

a) Determine the value of $\dfrac{\det \mathbf{J}^{\max}}{\det \mathbf{J}^{\min}}$ for the element, and comment on the expected accuracy.

b) Locate (calculate the x and y coordinates) the Gauss points for the 2×2 integration rule.

c) Use the 2×2 Gauss quadrature to evaluate the integral

$$I = \iint_A x^2 y dA$$

where A is the area of the element.

Q4. This question is related to your lab work.

a) The truss that was analyzed in ***Q2, Lab#1*** with the help of the bar element was solved again in ***Q1, Lab#4*** by using the beam element. In your opinion, did the results justify the afford put into replacing the element type? Would you recommend reanalyzing the water tower structure from ***Q3, Lab#1*** with the beam elements? How can you tell whether such a new analysis is really necessary?

b) Glass sticks partially submerged into hot water are prone to cracking. Based on the thermal stress analysis in ***Q3, Lab#3*** explain why the cracks *above* the water level should be orientated differently than the cracks *below* the water level.

c) Can the element *shell63* be used for the analysis of ***Q1, Lab#2***?
Can ***Q3, Lab#4*** be analyzed by using *plane42* instead of *shell63* to model the table, and by using *link8* instead of *beam4* to model the legs? Explain.

d) Write the ANSYS code that would solve the following problem:

6.1. The open-ended steel cylinder shown in Fig. P6.1 is subjected to an internal pressure of 1 MPa. Find the deformed shape and the distribution of principal stresses.

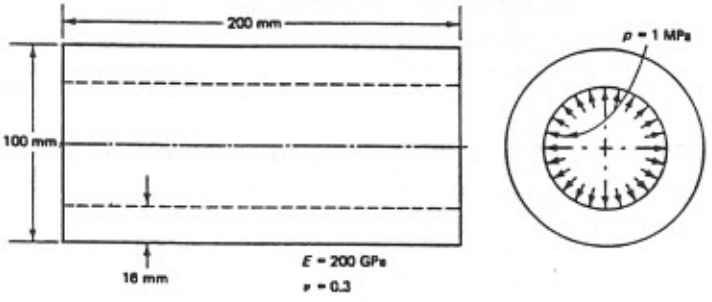
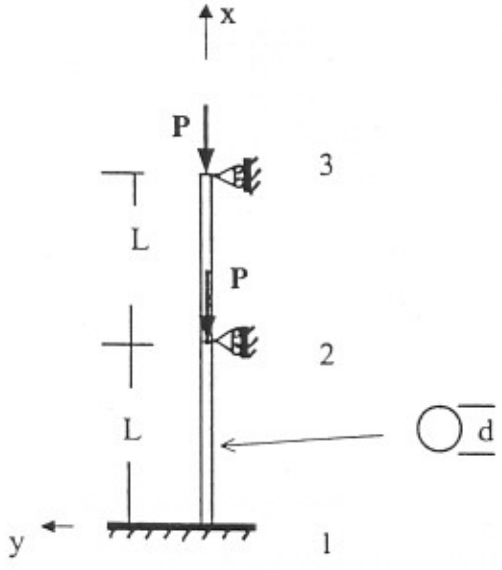


Figure P6.1

Q5. Use two beam elements to determine *the safety factor against buckling* for the post loaded by two forces **P**=20,900*N* as shown. Sketch the buckling modes obtained. Comment on accuracy of the two-elements solution.



L=1000*mm* d=20*mm* E=200GPa

THE END

Merry Christmas and Happy New Year !!!