UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Open-book examination
Answer **four** questions only

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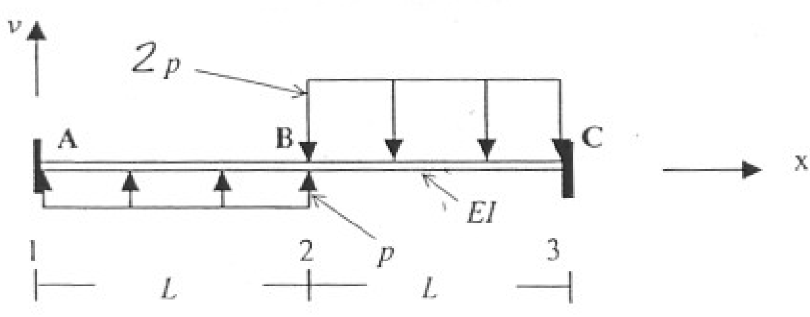
Q1. Use *two* elements and consistent force vector to solve the beam fixed at **A** and **C** and loaded as shown.
Determine, in terms of *EI*, *L* and *p*:

a) The deflection and slope at **B** (sketch the shape of the deflected beam).
b) The reactions at **A**.
c) The bending moment and the shear force for the first element (nodes 1-2) at **B**.
d) Locate the maximum deflection (absolute value) of the beam.

How would your results obtained in (**a-d**) be affected if *four* equal size beam elements were used?

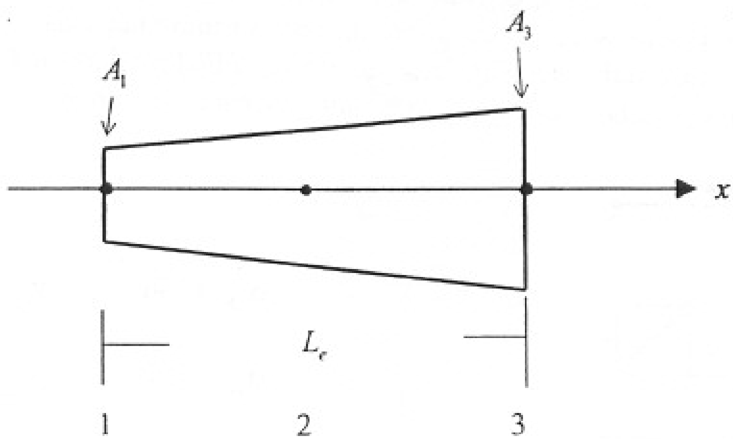


Q2. The stiffness matrix for the bar element of *constant area* with the *quadratic* shape functions was derived in class as:

$$K = \frac{AE}{3L_e}\begin{bmatrix} 7 & -8 & 1 \\ -8 & 16 & -8 \\ 1 & -8 & 7 \end{bmatrix}$$

a) Write the integral representing the term K_{ij} of the stiffness matrix of the bar element with the *quadratic* shape functions and the *area varying* according to:

$$A(x) = A_1 \frac{x_3 - x}{L_e} + A_3 \frac{x - x_1}{L_e}$$



b) Apply the Gauss quadratures to determine K_{11} (in terms of A_1, A_2, L_e, and E) of the stiffness matrix for the element in **(a)** using:

-one Gauss integration point,

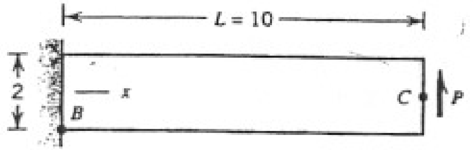
- two Gauss integration points.

Explain how your result would change if three Gauss integration points were used.
Assume that node 2 is in the middle of the element.
Hint: Use the natural coordinate

Q3. Give brief answers:

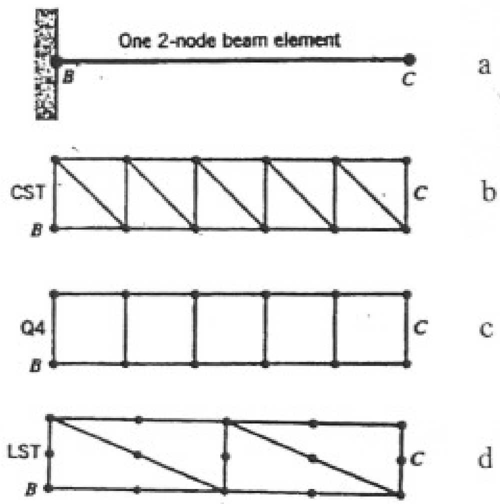
a) For the problem shown the following results are provided by 2-D theory of elasticity:

$\sigma_{xB} = 300MPa$ -stress at B, $v_C = 1.031mm$ - vertical displacement at C



Dimensions in mm, $E = 10^4 MPa$ $\nu = 0.3$, P=2N
Thickness=0.1mm

The problem was solved using the meshing below and applying: one beam element, constant strain triangular elements, quadrilateral elements, and linear strain elements respectively. Different results were obtained for different meshing patterns. Considering the properties of the elements used, match the FEM models on the left side with the results the results listed on the right side. Explain your answer.



1) $\sigma_{xB} = 254$ $v_C = 0.987$

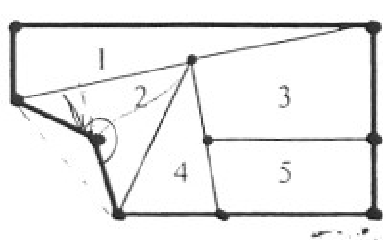
2) $\sigma_{xB} = 200$ $v_C = 0.693$

3) $\sigma_{xB} = 71$ $v_C = 0.264$

4) $\sigma_{xB} = 300$ $v_C = 1.031$

b) Write the formula for the consistent *mass* matrix (in the integral form) for the bar element in **Q2**.
- What is the size of this matrix?.
- How many Gauss points would be required to obtain *exactly* the term M_{11} of the matrix?

c) The meshing shown is to be used with the isoparametric quadrilateral elements.
- Is the meshing acceptable (explain!)?
- How would you modify it (using 5 elements) in order to obtain better results?

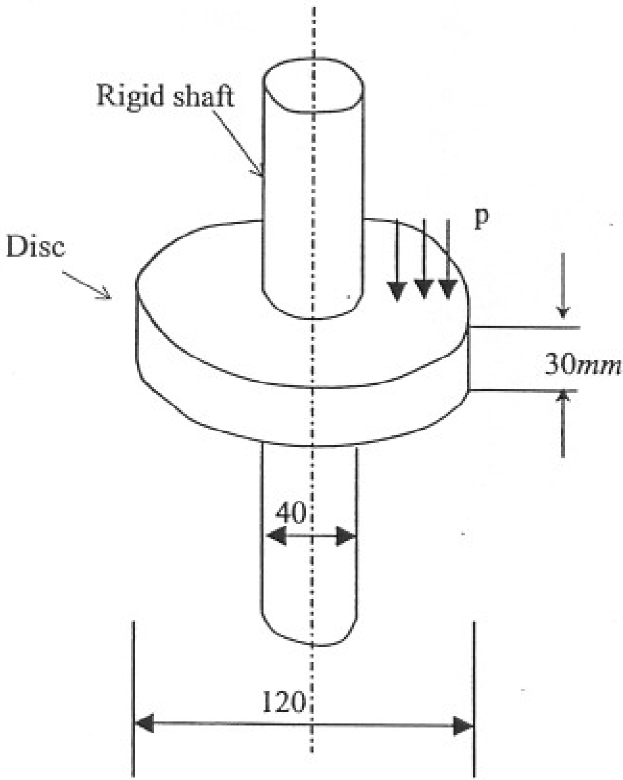


Q4. This question is related to your lab work.

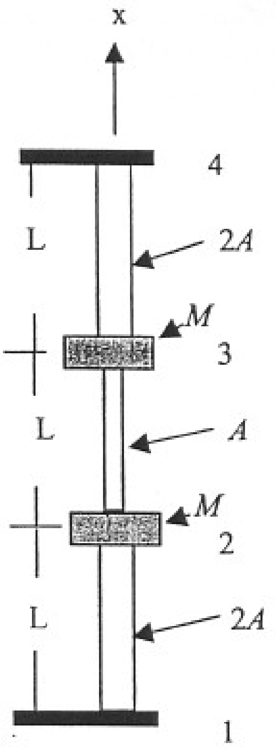
a) Assume that 2-D beam element (**beam3**) is used in *Lab1,Q2* (instead of **link1**) and in *Lab4, Q1* (instead of 3-D **beam4**) to analyze the plane frame structure considered. How will it change the results in both these labs?

b) Explain how you would check whether the meshing used in *Lab3, Q2* were acceptable.

c) A circular disc made of plastic (E=20*GPa* ν=0.3) is attached to a steel shaft as shown. A constant pressure p=2MPa is to be applied to the upper surface of the disc. You are to examine the deformation and stresses in the disc. Write the ANSYS prep file that would solve the problem. Assume that the shaft is rigid.



Q5. Use *three* linear bar elements and the consistent mass matrices to determine *two* frequencies of free *axial* vibrations of the structure with two concentrated masses as shown. Sketch the corresponding vibration modes.



Assume: $E = 70GPa$

$\rho = 2800\, kg/m^3$

$A = 10^{-3}\, m^2$

$L = 2m$

$M = 5.6kg$

THE END

Merry Christmas and Happy New Year !!!