UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
**ME 450.3 FINITE ELEMENT ANALYSIS**
FINAL EXAMINATION

Time: 3 hours
Open-book examination
Answer **three** questions only

December 2003
W. Szyszkowski

---

**Q1**. The plane-stress quadrilateral element of the shape shown is to be used in the FEM model.

**a)** Determine the Jacobian matrix $J$ and find the ratio $\frac{\det J^{min}}{\det J^{max}}$.

**b)** The term $K_{11}$ of the stiffness matrix for the element is defined as:

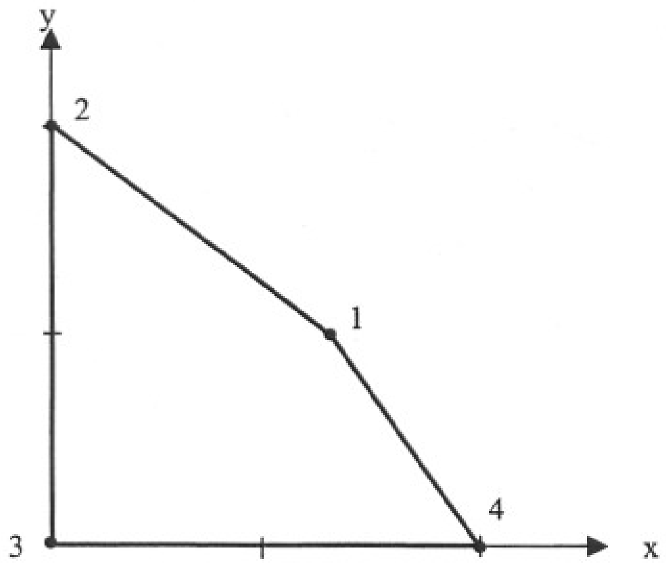
$$K_{11} = \frac{Et}{1-\nu^2} I \text{ where } I = \int_{-1}^{1}\int_{-1}^{1} (b_1^2 + 0.35c_1^2) \det J d\xi d\eta$$

$$\text{and where } b_1 = \frac{1-\eta}{4\det J} \text{ and } c_1 = \frac{1-\xi}{4\det J}$$

Use the $2\times 2$ Gauss integration to calculate $I$, and comment on the effect of $J$ on accuracy of the result. The exact value is $I_{exact} = 1.1275$.

**c)** Locate the Gauss points on the real element, and briefly explain how they are used to calculate the element's stresses at the nodes.

**d)** Would the ANSYS program accept the element of this particular shape? Justify your answer.




| node | x(mm) | y(mm) |
|---|---|---|
| 1 | 1.25 | 1 |
| 2 | 0 | 2 |
| 3 | 0 | 0 |
| 4 | 2 | 0 |

**Q2.** Apply <u>symmetry</u> and *two* beam elements to determine:

**a)** The maximum deflection in section AB.

**b)** The maximum deflection in section BC.

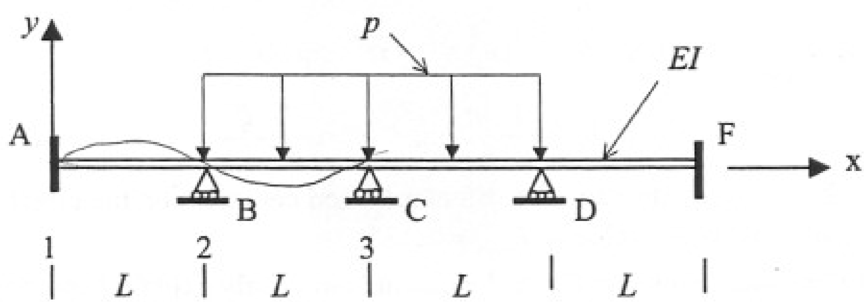
**c)** Calculate the reactions at A, B and C.

**d)** Determine the bending moment and the shear force for the elements.
Plot **M** and **V** for the whole beam (use the ANSYS sign convention).

**e)** Which of the results (**a-d**) will be affected if the problem is solved using 10 elements?

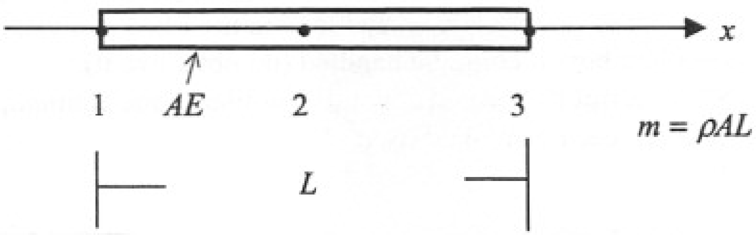
**f)** Assume that BEAM4 (not BEAM3!) element from ANSYS is used to solve (**a-d**).
Write the ANSYS commands to represent the boundary conditions.

Solve the problem in terms of *L*, *EI*, and *p*.



**Q3**. Solve the following problems:

**a)** For the *quadratic* bar element (the *quadratic* shape functions) of *constant area* obtain the consistent mass matrix *M* in terms of the element's mass *m*, if the exact values of integrals $I_{ij} = \int_{-1}^{1} N_i N_j d\xi$ are: $I_{11} = I_{33} = 4/15$ , $I_{12} = I_{23} = 2/15$, $I_{13} = -1/15$ , $I_{22} = 16/15$

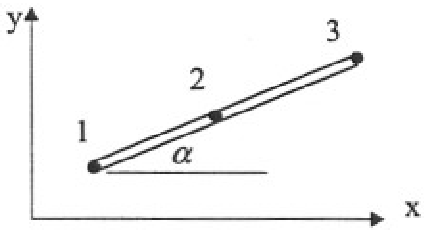



**b)** Apply the Gauss integration (use enough Gauss points to get exact results) to verify the value of $I_{22}$

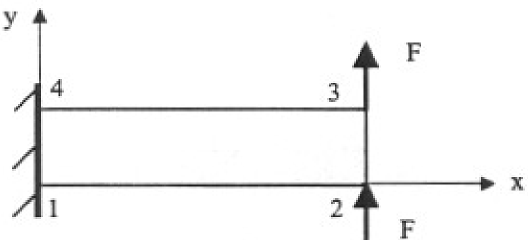
**c)** The quadratic bar element from (**a**) is to be used in the 2-D global coordinate system x-y. Write the transformation matrix **L** (in terms of the direction cosines *l* and *m*) so that the stiffness matrix in this system can be expressed as:

$$K_{global} = \mathbf{L}^T K_{local} \mathbf{L}$$

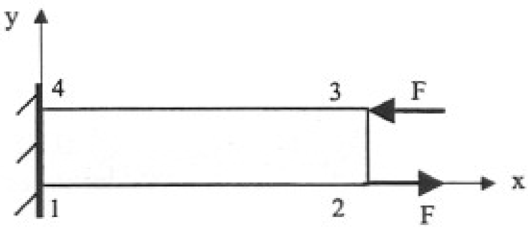
where $K_{local}$ is the stiffness for the configuration shown in (**a**).
Assume that $\alpha$ is given.




**d)** If *one* quadrilateral element (such as PLANE42 from ANSYS) is used as shown, then the results for the loading (1) are about 50 less accurate then for the loading (2).
Explain why.




loading (1)




loading (2)

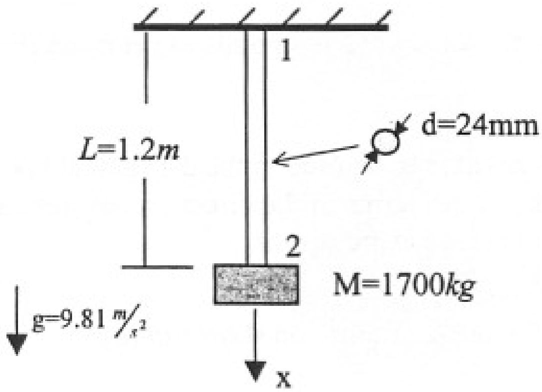
**Q4.** A 1700*kg* mass is attached to the ceiling as shown.
You are to determine the lowest two frequencies of vibrations.
Use one element and two nodes as indicated to obtain:

**a)** The lowest frequency of the axial vibrations.

**b)** The lowest two frequencies of the flexural vibrations ignoring the effect of gravity.

**c)** Is the gravity important for this problem?
Which frequency will be affected and how much (give detailed answers)?
Show how the effect of gravity can be included in the FEM analysis. Formulate the problem and explain how it could be handled (do not solve it).

**d)** Write the ANSYS script that would solve the problem (one element, two frequencies).
Briefly comment on each command used.




$\rho = 7800\ kg/m^3$

E=200GPa

THE END

Merry Christmas and Happy New Year!!!