UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Closed-book examination
3 double-sided sheets of formulas are allowed
Answer **three** questions only

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Q1. Apply *one* element of length L to evaluate the deformations, stresses, vibrations and stability of the beam shown in Figure 1.

a) Determine the slope at node 1 and the reactions due to gravity load p. Calculate the location of maximum absolute deflection, and sketch the deformed configuration of the beam. Also, for comparison, sketch the displacement plot that would be obtained from ANSYS if one **beam3** element was used.

b) Use the reaction results to determine the nodal $\boldsymbol{M}$ and $\boldsymbol{V}$. Plot the $\boldsymbol{M}$ and $\boldsymbol{V}$ diagrams using the ANSYS sign convection (as if obtained from ANSYS, and one **beam3** element). Calculate the maximum bending stress, and the maximum average shear stress in the beam.

c) Use the geometrical stiffness matrix to solve the buckling problem if a horizontal force **H**=10,000N is applied. Determine the safety factor for stability. Sketch the corresponding buckling mode.

d) Use proper mass matrices to determine the frequency of flexural free vibrations, and then the frequency of axial free vibrations. Sketch the corresponding modes of vibration.

e) How would your results in **a-d** be affected if the beam was meshed with say 20 elements?

Given: E=200GPa, $\rho = 7800 \, {}^{kg}/_{m^3}$, $I = \frac{\pi}{64} D^4$

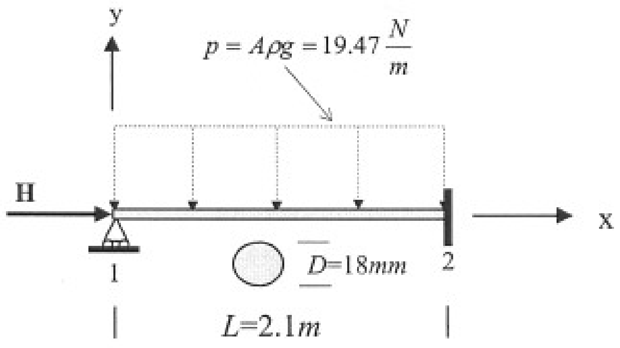


Figure 1

Q2. (**a**) The structure shown in Figure 2' was analyzed in *labs #1*, *#4* to evaluate:

- The maximum vertical displacement.
- The maximum stress.
- The safety factor for stability.

It was modeled by either **link1**, or **beam4** ANSYS elements. However, the structure may also be modeled by using elements **link8**, or **beam3**.

Explain in detail pros & cons of applying the above elements.

In particular, describe how the results related to the structure's displacements, stresses, and stability will be affected, if the following element type is used:

1) **link1** - the 2-D bar element,
2) **link8** - the 3-D bar element,
3) **beam3** - the 2-D beam element,
4) **beam4** - the 3-D beam element.

Be as specific as you can when describing similarities or/and differences in the results obtained from the above elements.

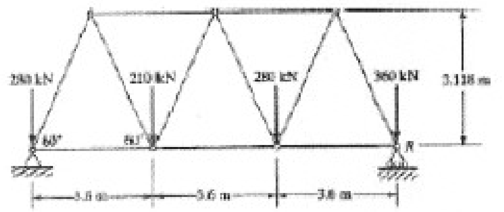


Figure 2'

(**b**) In the problem analyzed in *lab #4* (see Figure 2") elements **beam4** and **shell63** were used to model the legs and the top plate respectively.

1) Can **beam4** be replaced by **link8**, and **shell63** be replaced by **plane42** in this analysis?
2) Can the legs be modeled by **shell63** elements?
   Justify your answers to questions (1) and (2).
3) The structure has two planes of symmetry. How the calculated maximum displacements, maximum stresses, and the frequencies of free vibrations will be affected if only a quarter of the structure is modeled.

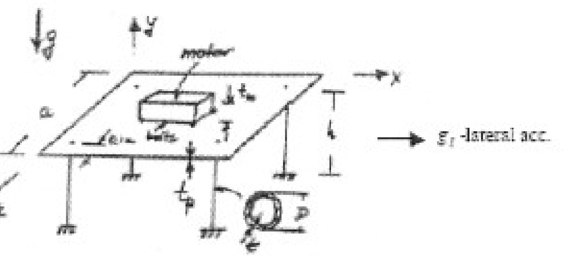


Figure 2"

Q3. (**a**) The fictitious moment at node 1 of the beam element is defined as

$$M_1^p = \int_{-1}^{1} p(\xi) \cdot N_2 \cdot \det J \cdot d\xi, \text{ where the shape function } N_2 = \tfrac{1}{2} L H_2,$$

and where $H_2 = \frac{1}{4}(1-\xi)^2(\xi+1)$.

Determine M_1^p for the distributed load p given as $p(\xi) = p_1 \frac{1-\xi}{2} + p_2 \frac{1+\xi}{2}$ (see Figure 3') by using: **1**) *two* Gauss integration points,
2) *three* Gauss integration points.

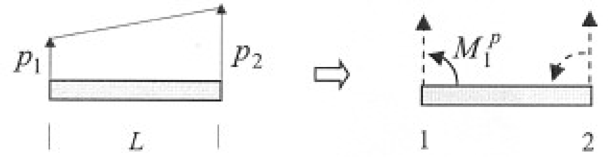


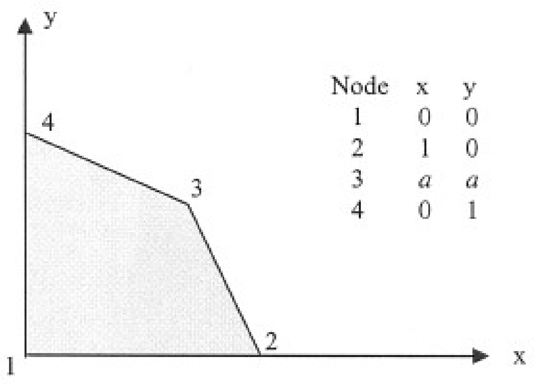
Figure 3'

Compare with the exact value $M_1^p = \dfrac{L^2}{60}(3p_1 + 2p_2)$ obtained from the direct integration.

(**b**) For the quadrilateral element shown determine:

1) The terms $\dfrac{\partial x}{\partial \xi}$, $\dfrac{\partial x}{\partial \eta}$, $\dfrac{\partial y}{\partial \xi}$, and $\dfrac{\partial y}{\partial \eta}$ of the Jacobian J.

Show that $\det J = \dfrac{a}{4} - \dfrac{1-a}{8}(\xi + \eta)$.



Node	x	y
1	0	0
2	1	0
3	a	a
4	0	1

Figure 3"

2) For a=0.6 find the value of $\dfrac{(\det J)_{\max}}{(\det J)_{\min}}$, and comment on the meaning of this ratio.

Determine the x,y coordinates of G_2 (the Gauss point corresponding to $\xi = 1/\sqrt{3}$ and $\eta = -1/\sqrt{3}$).

3) Explain in detail why the element becomes unacceptable if $a \le 0.5$ (use the cases a=0.5 and a= 0.25 as illustrations, for example).
What would be minimum value of a to be acceptable by ANSYS without warning?

Q4. Explain how you would set up the ANSYS model for the cylindrical pressure vessel of diameter D and flat closures as shown in Figure 4. The stresses in the vessel that contains a fluid of temperature T (assume this temperature for the inside surfaces) and under uniform overpressure p are to be analyzed.
Also, you must make use of the plane of symmetry.

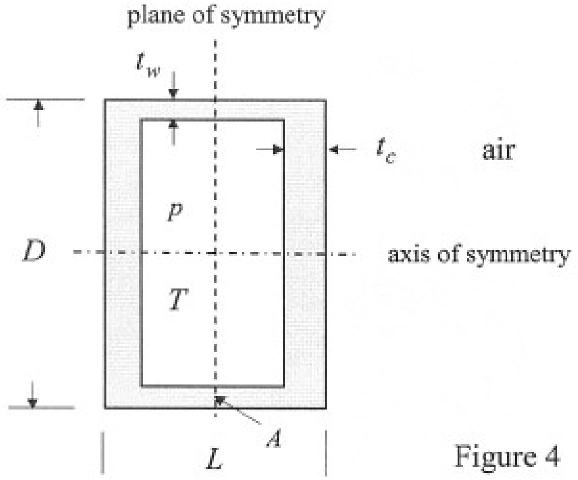


Figure 4

Assume that E (Young modulus). ν (Poisson's ratio), k (conductivity), h (convection to air) are given.

Also, assume: $T_{air} = 20^0 C$

p=20MPa, T=200$^0 C$
D=200mm, L=100mm
t_c = 30mm , t_w = 10mm

a) List the types of elements you would use.
b) Select the keypoints and identify their location. List your selection.

K_i	x_i	y_i
1	......	.....

c) Define and sketch the areas to be meshed.
d) Describe in detail the boundary conditions for the thermal problem.
e) Describe in detail the boundary conditions and the loading for the stress problem.
f) At point A the postprocessor renders the stresses: ***sx***=10, ***sy***=100, ***sz***=200, ***sxy***=0.
Explain the meaning and sketch the directions of these stress components.
g) What will be the value (and the meaning) of the stress denoted as ***seqv*** at point A?

* For (**d**) and (**e**) you may list the lines (or nodes) to which particular BC/loads are applied, no strict ANSYS commands are required.

THE END
Merry Christmas and Happy New Year!!!