UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Closed-book examination
3 double-sided sheets of formulas are allowed
Answer **three** questions only

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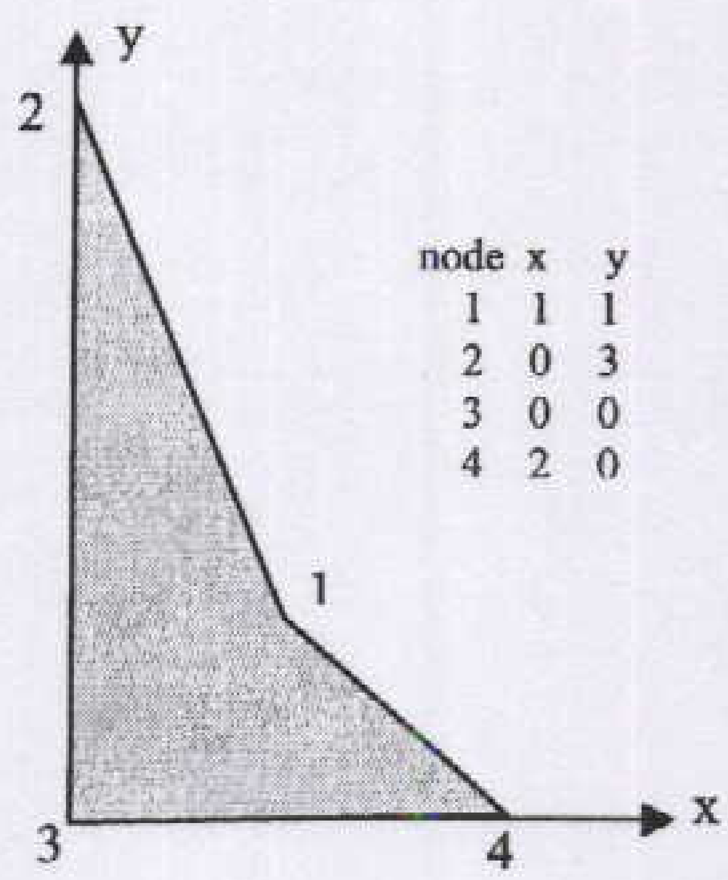
Q1. Provide detailed explanations and/or derivations as requested.

a) The element and nodal stresses were calculated in most of your assignments problems. Explain the main differences between these two types of stresses, and briefly explain how they are determined. Use the quadratic bar element and the plane stress CST element to illustrate your answer.

b) For the quadrilateral element shown:

- Obtain the x- and y-coordinates in terms of the ξ - and η -coordinates of the master element. Determine the origin and orientation on the ξ - η system and sketch it on the real element.
- Obtain the jacobian matrix (J), and then the function defining *detJ*.
- Determine the minimum and maximum values of *detJ* (indicate the corresponding locations).

Briefly explain why the shape of this element is not acceptable and how would you modify it to make it acceptable.



c) The buckling equation for beams is given in the form:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right)-\underline{\frac{d}{dx}\left(n\frac{dv}{dx}\right)}=0$$

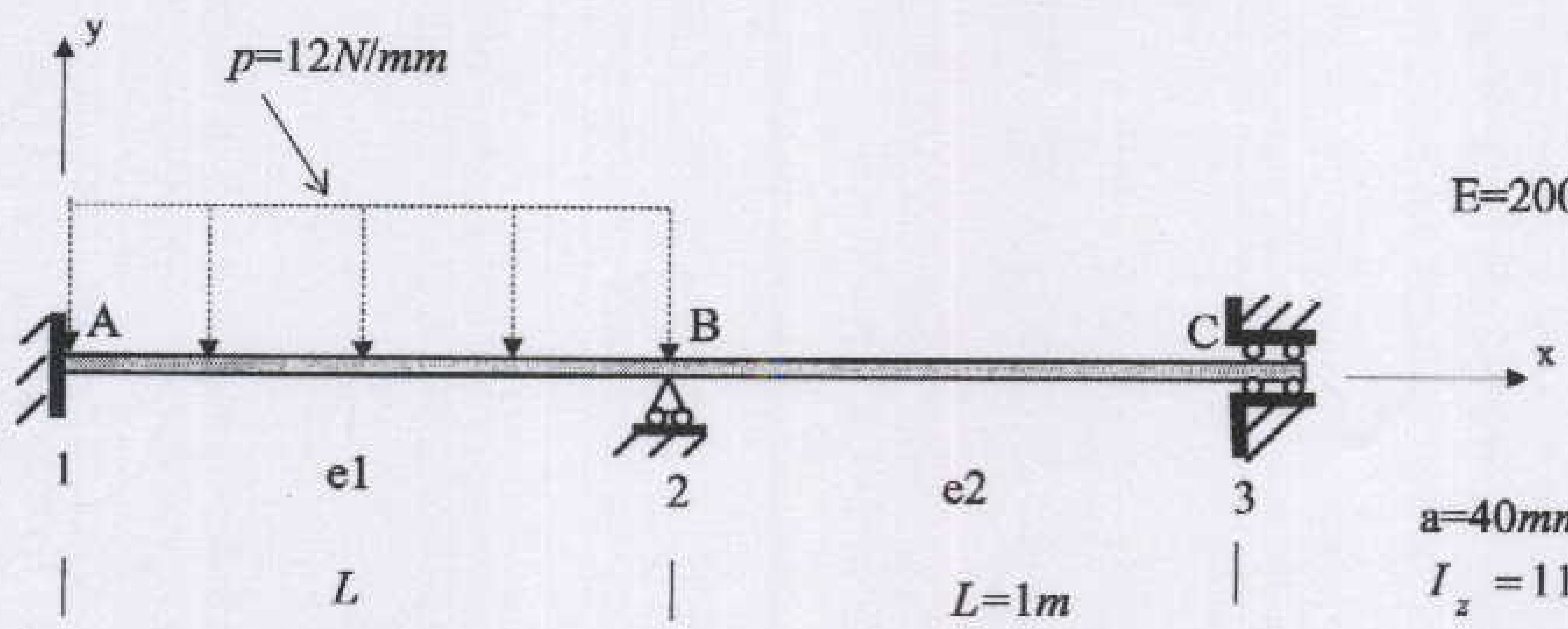
- Use Galerkin's method (and integration by parts) to show how the underline term can be converted into the geometric stiffness matrix defined as $K_{ij}^{g}=\int_{L_e}\frac{dN_i}{dx}n\frac{dN_j}{dx}dx$ (this matrix is used in the buckling analysis).

- If the natural coordinate is substituted then $K_{11}^{g}=\frac{2n}{L_e}\int_{-1}^{1}\left(\frac{dN_1}{d\xi}\right)^2 d\xi$ where $N_1=(2-3\xi+\xi^3)/4$.

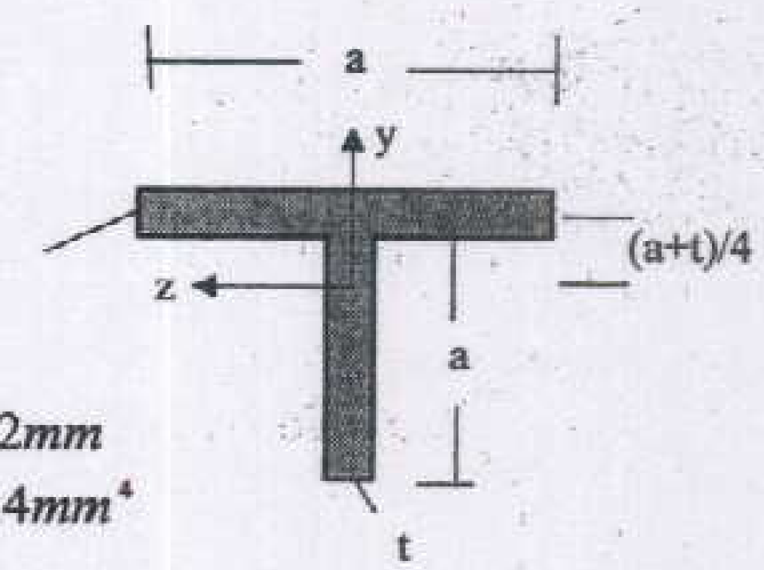
Use the <u>necessary</u> number of Gauss points and the <u>Gauss quadratures</u> to obtain the exact formula for K_{11}^{g} in terms of n and L_e.

Q2. Apply <u>two</u> elements of equal length to analyze the beam shown (note that the loading is in the x-y plane, and the slope and vertical displacement at C are zero).

1) Determine the slope and then the vertical reaction at B (node 2).
2) Determine the vertical deflections at the midpoints of sections AB and BC. Sketch the beam's deflected configuration.
3) Which of the results in (1) and (2) would be affected if the beam was modeled by 20 elements? Sketch the displacement plot that would be obtained from ANSYS if <u>two</u> *beam3* elements were used, and then if <u>twenty</u> *beam3* elements were used.
4) Obtain the nodal values of M and V (the bending moment and shear force) for each element. Use the ANSYS plotting convention to sketch the M and V diagrams for the whole beam. How these plots be affected if the beam was modeled by 20 elements?
5) Use the M plot obtained in (4) to determine the maximum <u>compressive</u> and maximum <u>tensile</u> stresses in the beam due to bending.



E=200GPa



$a=40mm$, $t=7.2mm$

$I_z = 119{,}846.4 mm^4$

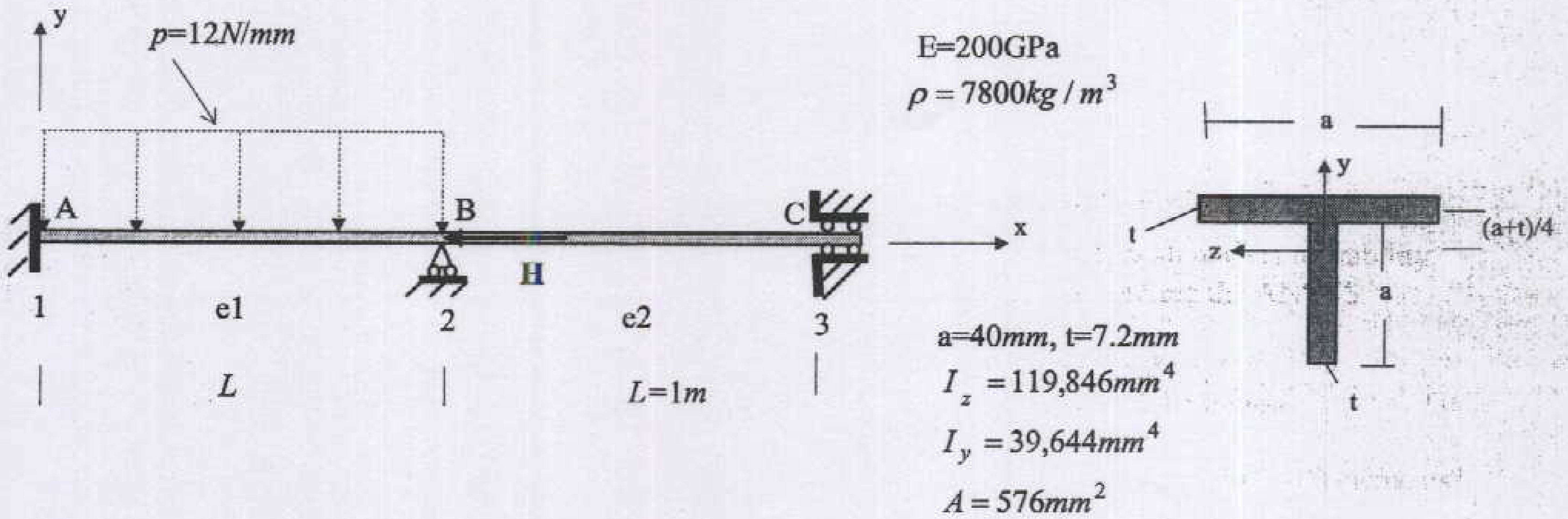
$I_y = 39{,}644.2 mm^4$

Q3. The beam from **Q2** is to be analyzed for stability and vibrations.
Assume that at B and C the displacements in the z-dir are also zero (and roty=0 at C).

a) Determine whether a horizontal force **H**=240k*N* applied at B as shown is safe with respect to stability.
A preliminary modeling of the beam by two *beam3* elements (as in **Q2**) and running the ANSYS buckling procedure rendered the lowest magnitude of *fact*=5.992.
- Formulate and solve the buckling problem by using two beam elements to verify the above result. Sketch the shape of the corresponding buckling mode. Would the lowest magnitude of *fact* be affected if the beam was modeled by 20 *beam3* elements?
- What would be the lowest magnitude of *fact* if the beam was modeled with two *beam4* (3-D) elements?
- Which of the ANSYS *beam* elements (*beam3* or *beam4*) would you recommend, and why?

b) Use two elements to determine the lowest frequency of *flexural* vibrations and then the lowest frequency of *axial* vibrations.

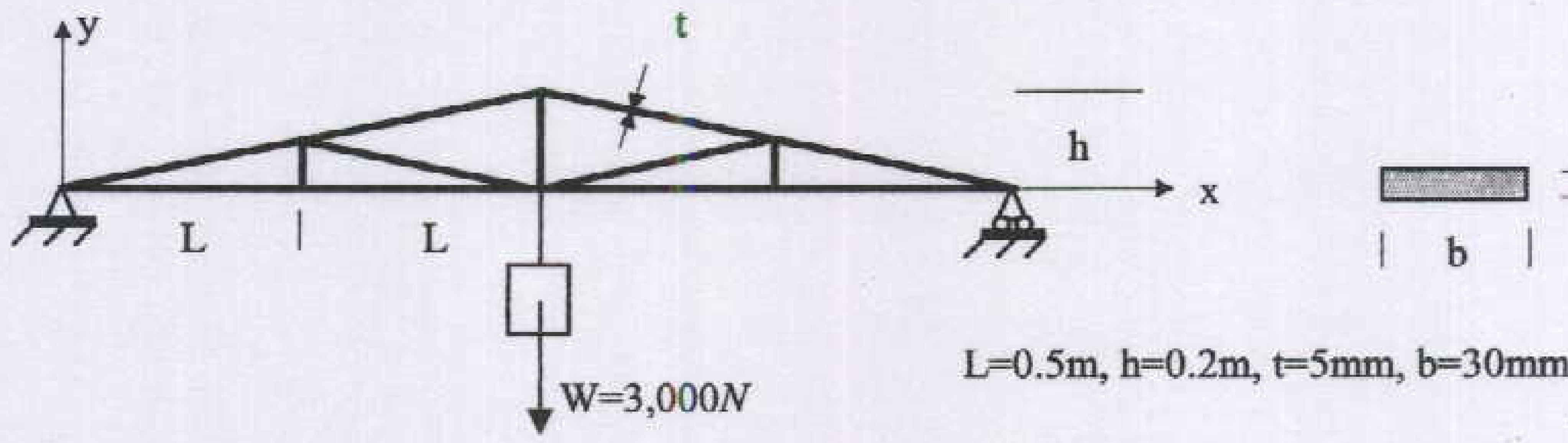
c) It is suggested to model the problem with the *plane42* or *shell63* elements.
What is your opinion on using these elements? In particular, what new or better results should be expected if you decide to use them? Explain briefly.



Q4. Provide detailed explanations of the ANSYS elements, routines and code as requested.

a) You are to verify whether weight W can be safely supported by the steel (E=200*GPa*, $\sigma_Y = 400MPa$) structure shown (note the symmetry). All the members are welded at the joints.

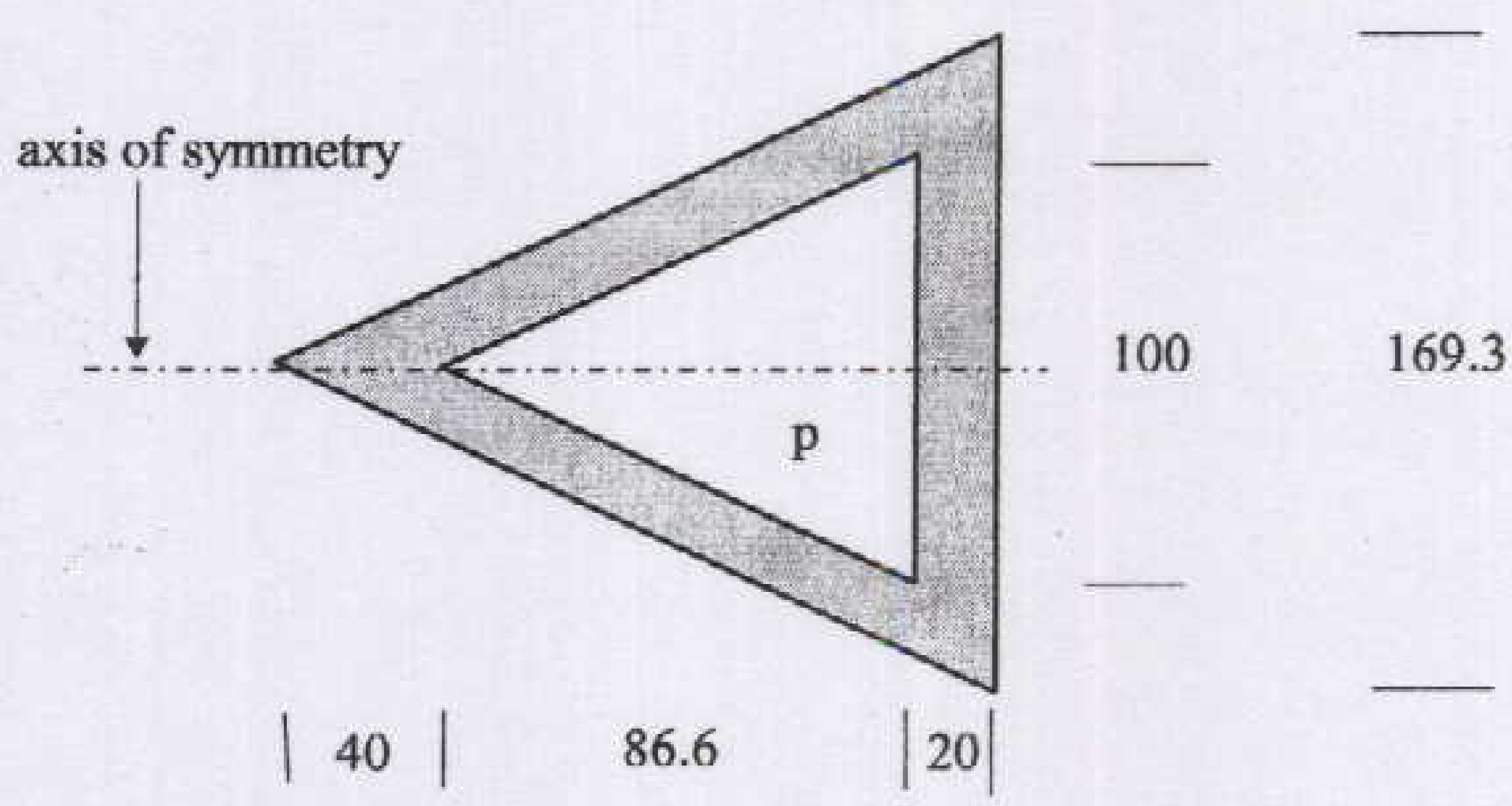
1) Select the element type that, in your opinion, suits the best for modeling the problem, briefly justify your selection.
2) Write the /PREP code that provides the complete information on the model (geometry, nodes, elements, material, load, and boundary conditions).
3) Describe the solution routines you would use, the results expected and how they could be obtained from the /POST (no detailed code required in this part of the question).



b) The conical axisymmetric steel capsule to contain a fluid under internal pressure p=10MPa is designed as shown. You are to evaluate the capsule's stresses and deformation.

1) Write the complete /PREP code to model the capsule. Justify the type and size of elements used.
2) Explain what results you would read in the /POST, and how the solution's accuracy could be verified.

What design's modifications, if any, would you suggest to improve strength of the capsule?



E=200*GPa*, $\sigma_Y = 400MPa$

ν=0.3

All dimensions in *mm*.

THE END