UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
FINAL EXAMINATION

Time: 3 hours
Closed-book examination
3 double-sided sheets of formulas are allowed
Answer **three** questions only

December 2008
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Q1. Provide detailed explanations and/or derivations as requested.

a) Determine the *number* of integration points that is needed to obtain *exactly* by means of the Gauss integration the following:

- The terms of the mass matrix for the linear bar element of constant area defined by $M_{ij}^{bar} = \int_{L_e} N_i A \rho N_j dx$
- The terms of the mass matrix for the beam element of constant area defined by $M_{ij}^{beam} = \int_{L_e} N_i A \rho N_j dx$
- The components of the fictitious nodal force vector for the beam element defined by $F_i^p = \int_{L_e} p N_i dx$

if the transversal distributed load p is constant ($p = p_0$).

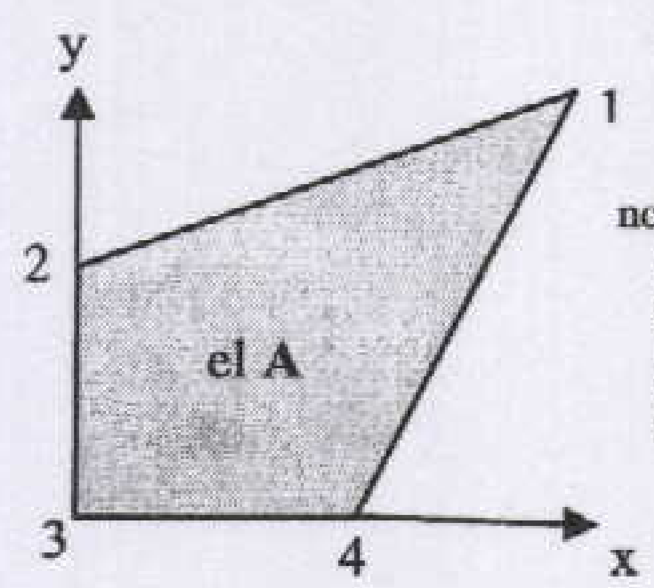
b) Apply 2 integration points and the Gauss quadratures to obtain M_{11}^{bar}, M_{11}^{beam}, and F_1^p (note that the Gauss integration should be performed on the master elements and that $F_1^p = V_1^p$).
Comment on accuracy of the results obtained.

c) For the quadrilateral element **A** below obtain:

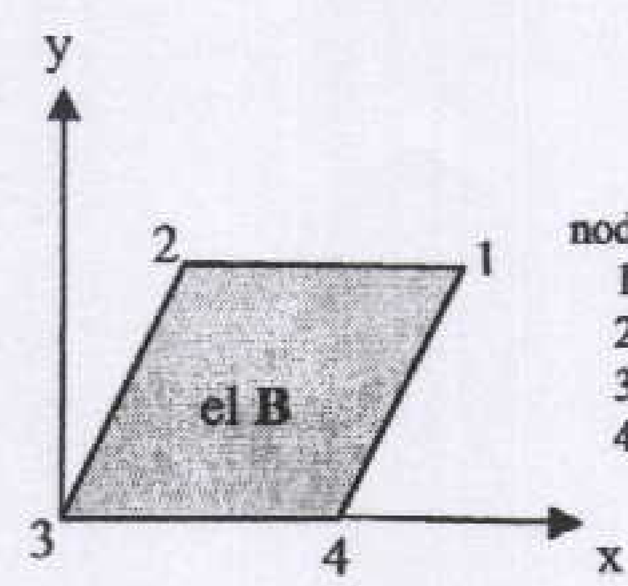
- The x- and y-coordinates in terms of the ξ - and η -coordinates of the master element.
  Determine the origin and sketch the orientation of ξ and η on this element.
- The jacobian matrix (**J**), and then the function defining *det***J**.

Briefly describe how *det***J** is related to accuracy of the 2×2 Gauss integration scheme typically used to obtain the terms of the stiffness matrix for such elements. In particular, explain why:

- Element **A** would be handled by ANSYS without any warning about its shape.
- Integrating ***K*** (stiffness matrix) for element **A** would be less accurate than integrating ***K*** for element **B**.



node	x	y
1	2	2
2	0	1
3	0	0
4	1	0

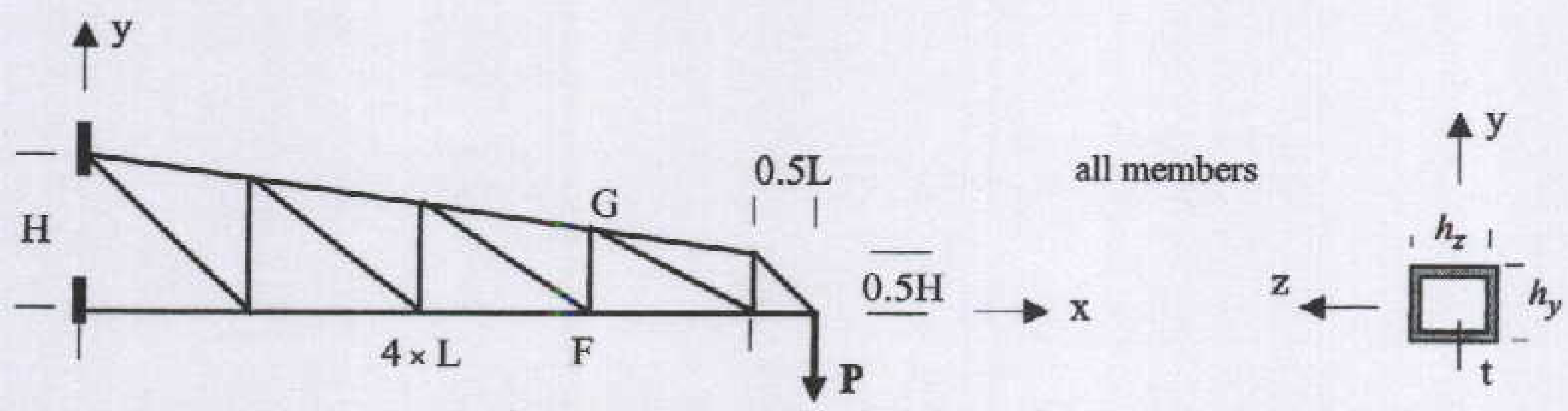


node	x	y
1	1.5	1
2	0.5	1
3	0	0
4	1	0

Q2. This question pertains to the ANSYS elements, routines and codes used in the labs and assignments.

a) The boom structure was analyzed for displacements, stresses, and stability by applying the *link1*, *beam3*, and *beam4* elements.

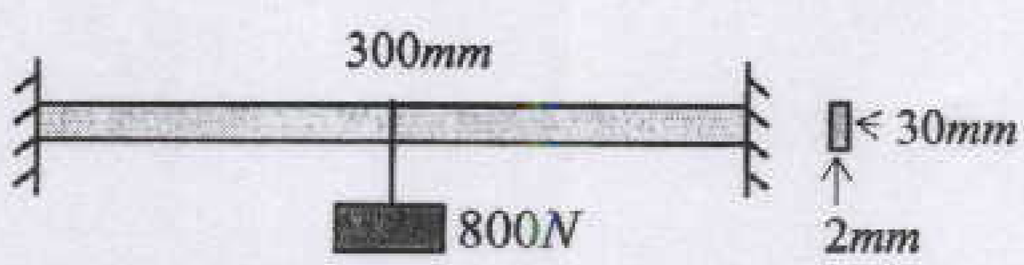
1. Finish the following statements (first copy each statement to your exam paper!):
   - Sufficiently accurate displacements can be obtained by using the elements ....
   - Sufficiently accurate stresses can be obtained by using the elements ....
   - Sufficiently accurate safety factor with respect to stability can be obtained by using the elements ....
2. Could *link1* be replaced by *link8* in this problem (with the same mesh pattern and nodes at the joints)? If yes, explain the code's modifications that would be required.
3. Could *link1* be used with the meshing as for *beam3* (5 elements per member and 4 extra nodes between the joints)? If yes, explain the code's modifications that would be required.



b) The 'ruler' problem was analyzed in the lab (and in class) for displacements, stress, and stability by using several different elements. For the problem shown below (identical structure, slightly different end supports), give your best recommendations on the meshing (element size, pattern, etc) if the following elements are to be used:

1. Element CST
2. Element LST
3. Element *plane42*
4. Element *plane82*
5. Element *shell63*
6. Element *beam3*

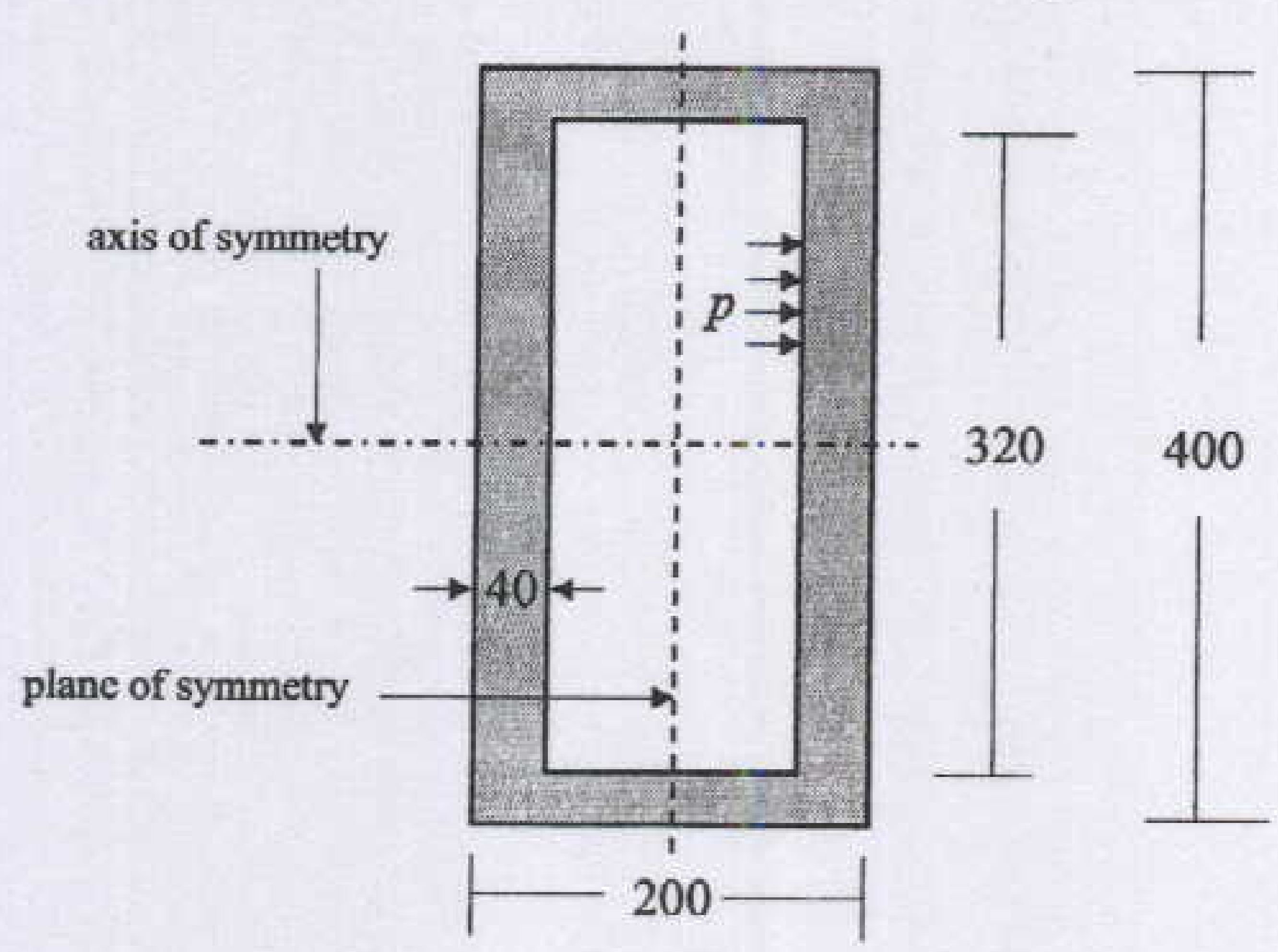
Comment on the expected accuracy of calculated displacements, stress, and stability, and also on the computational effort required for each case.



c) You are to evaluate the stresses and deformation in the cylindrical steel container under internal pressure of p=10MPa. Note the orientations of the axis of symmetry and the plane of symmetry.

1) Write the complete /PREP code to analyze the container (sketch your model and indicate the CS selected). Justify the type and size of elements used.
2) Explain how the solution's accuracy could be verified.

What design's modifications, if any, would you suggest to improve strength of the capsule?



E=200*GPa*, ν=0.3

All dimensions in *mm*.

Q3. The beam ABC supported as shown was loaded transversally and analyzed by applying the ANSYS' *beam3* element. You are given a report with the shear force (V) and the bending moment (M) plots obtained from the postprocessor for 2el and for 20el solutions. However, the load description is omitted for some reasons.

a) Determine the reaction at A, B, and C (show the values and directions).

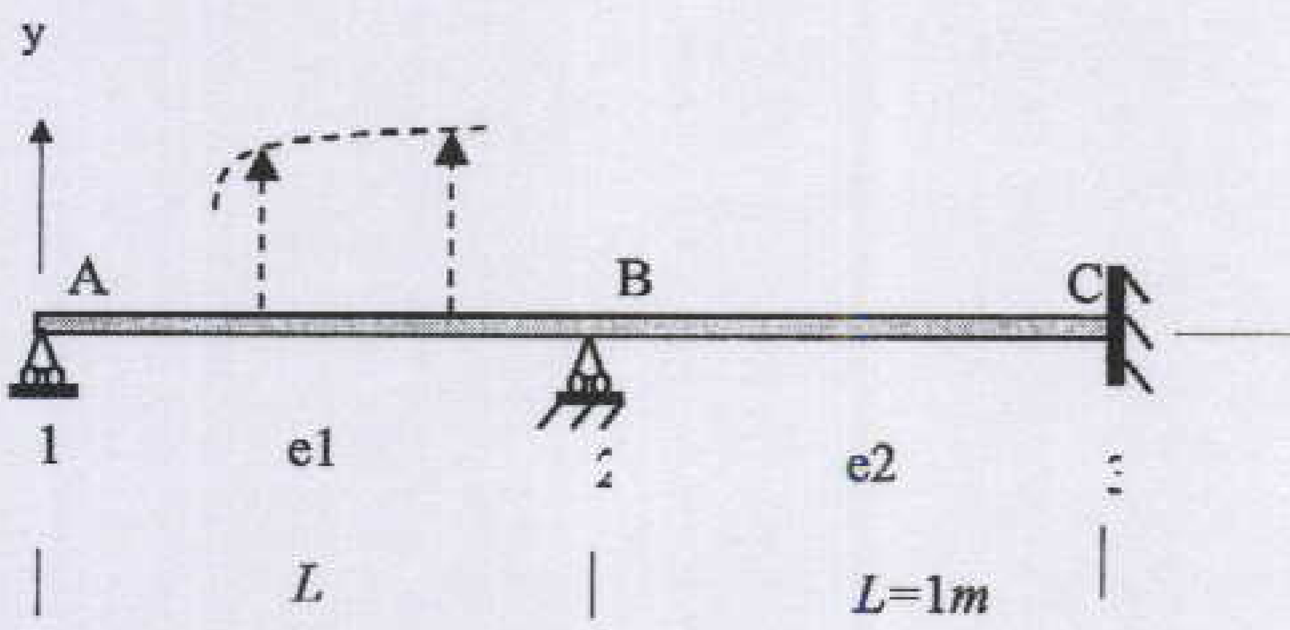
b) Determine the maximum tensile and the maximum compressive stresses for the 2el solution, indicate the location of these stresses.

c) Determine the maximum tensile and the maximum compressive stresses for the 20el solution, indicate the location of these stresses.

d) Determine the maximum shear stress due to **V**

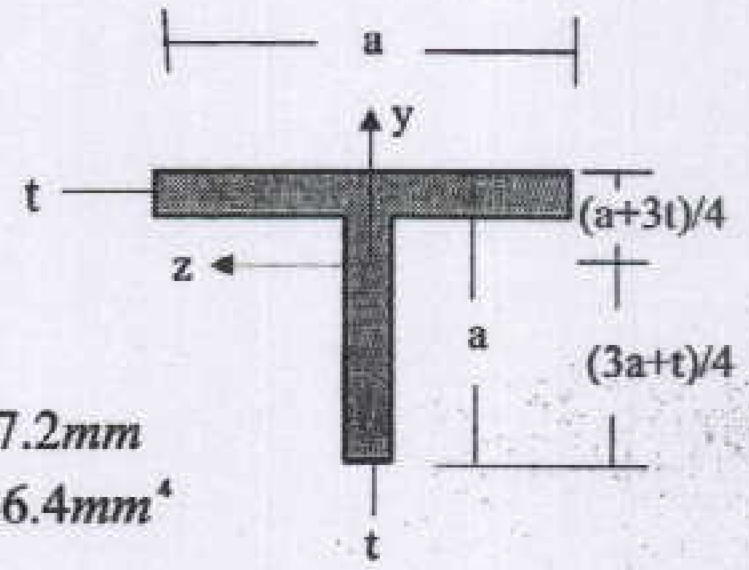
e) Sketch the **V** and **M** plots that would have been obtained for the 4el solution (elements of equal length).

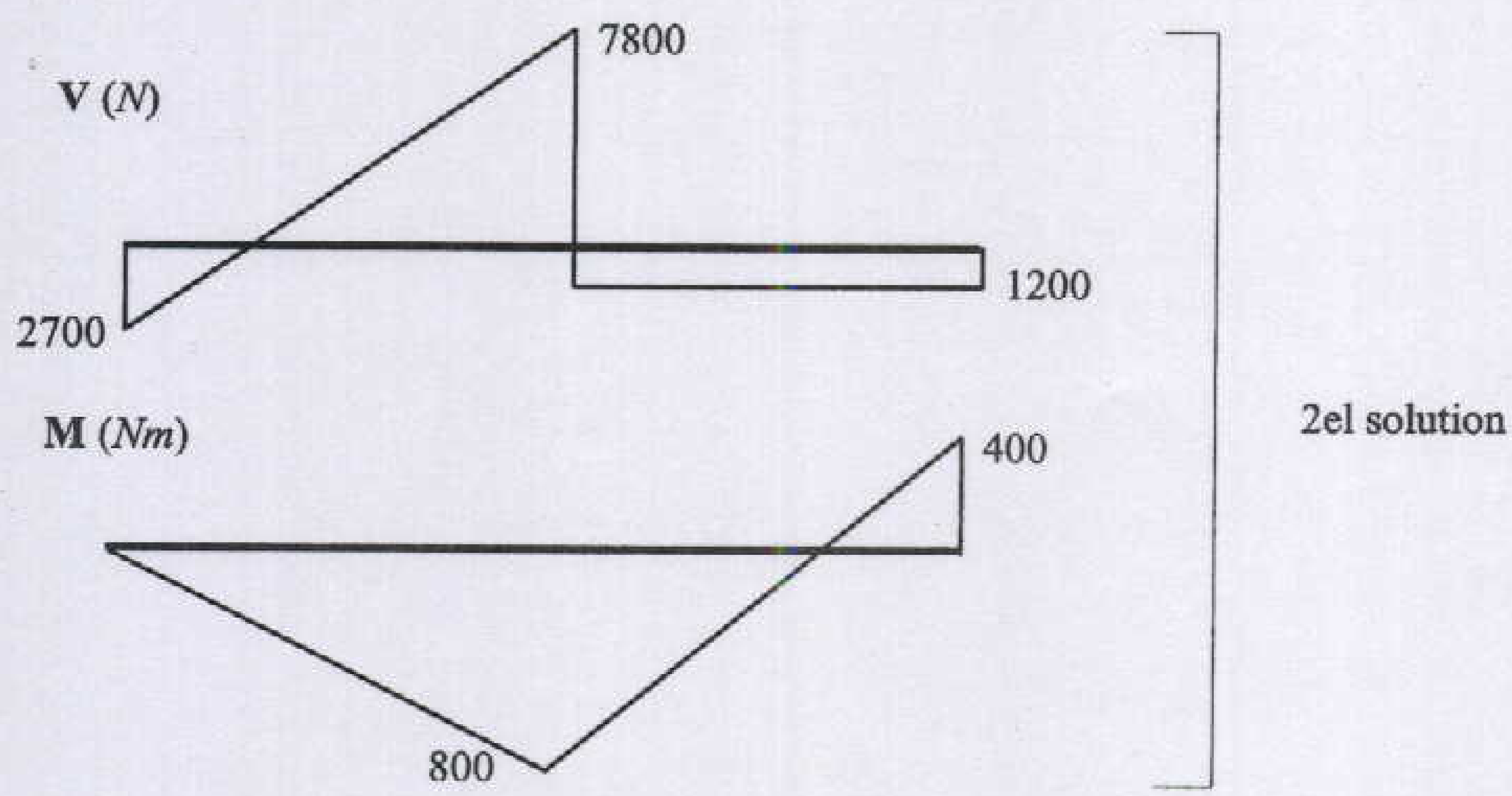
f) Recreate the loading applied, if possible.

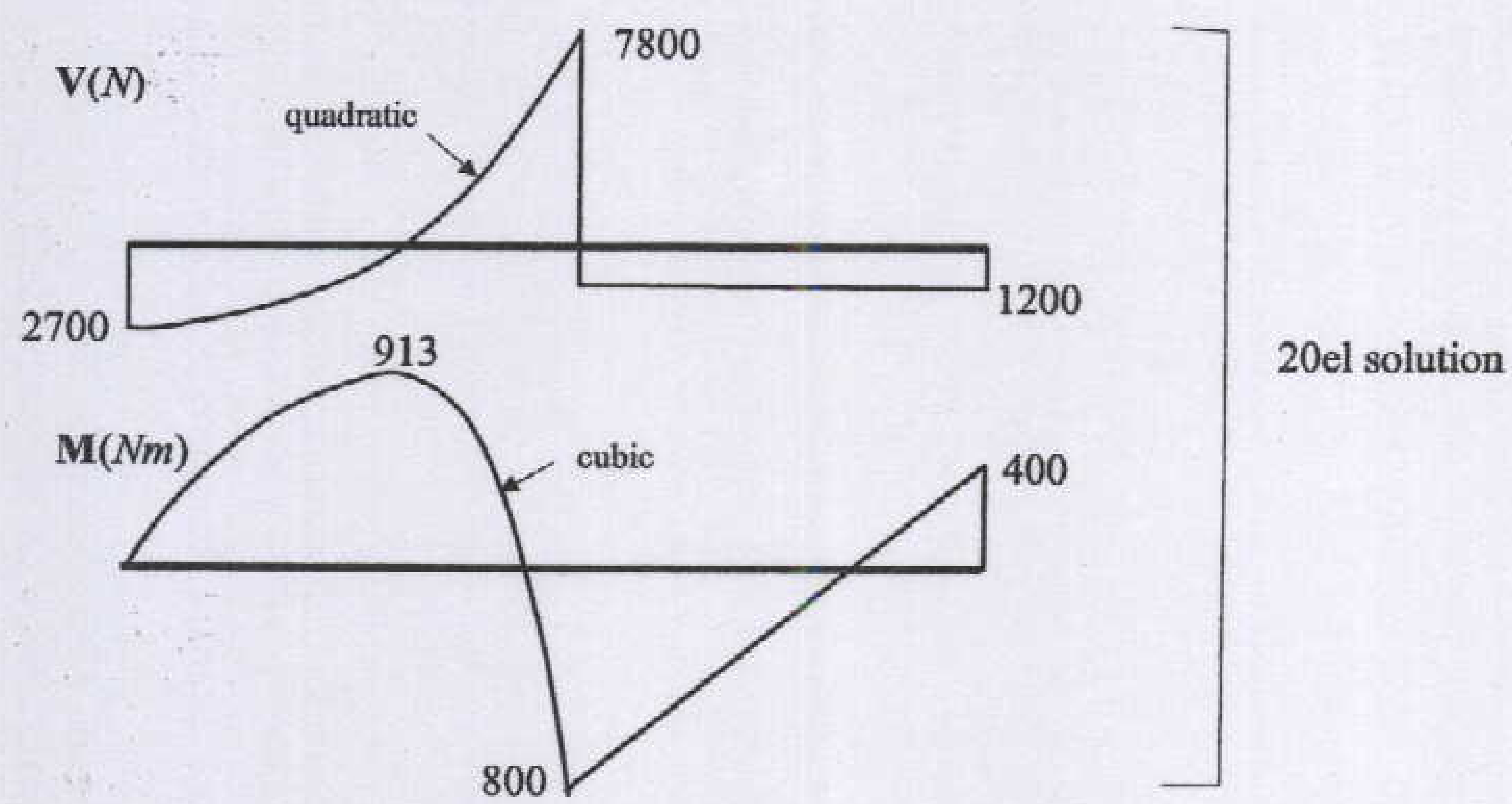


$a=40mm$, $t=7.2mm$

$I_z = 119{,}846.4mm^4$







Q4. The beam supported as shown is to be analyzed for stability.

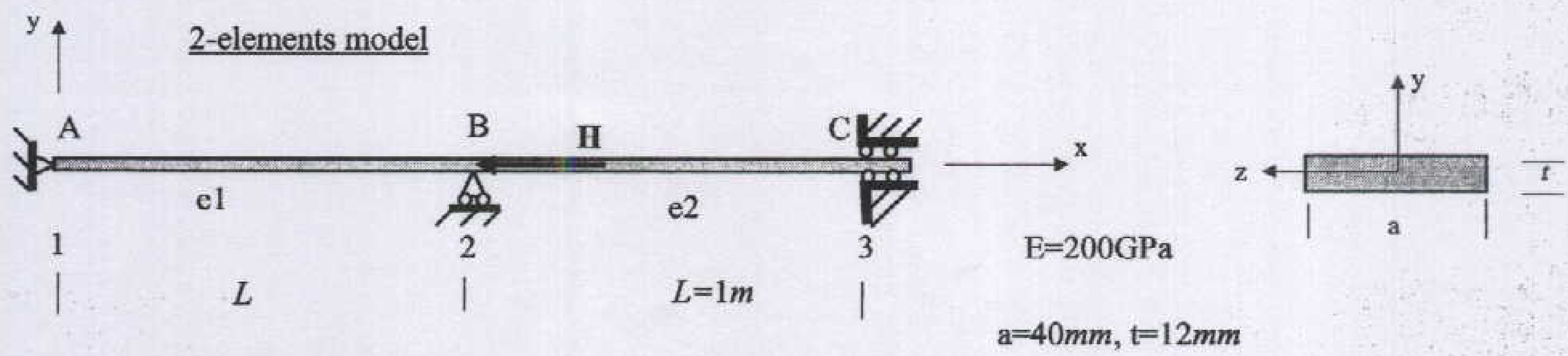
Note that the end C cannot rotate (but it's free to displace horizontally).

a) Use *two* beam elements to determine the safety factor with respect to stability if a horizontal force **H**=20*kN* is applied at B.
Determine and sketch the shape of two lowest buckling modes.

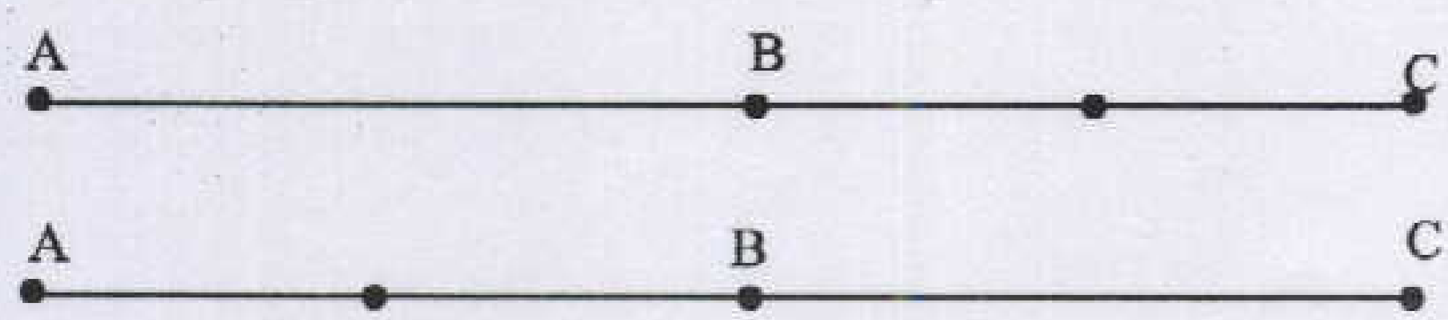
b) Write the corresponding ANSYS code (or describe the steps needed if you don't remember the detailed commands) to solve the 2-el problem above (*hint*: build the model by specifying nodes and elements).
Justify your choice of the element type (*beam3* or *beam4*?).
How would the ANSYS plots of the buckling modes compare with the modes obtained in (**a**)?

c) Two models consisting of 3 elements are considered to improve the accuracy. Would you recommend to use the model with 2 elements is section BC, or the model with 2 elements in section AB? Explain your answer.
Due to gravity the beam deflects vertically by 0.5*mm* in the middle of section AB if force **H**=0.
Explain how this deflection will change if **H**=20*kN* is applied (use the buckling results obtained in (**a**)).

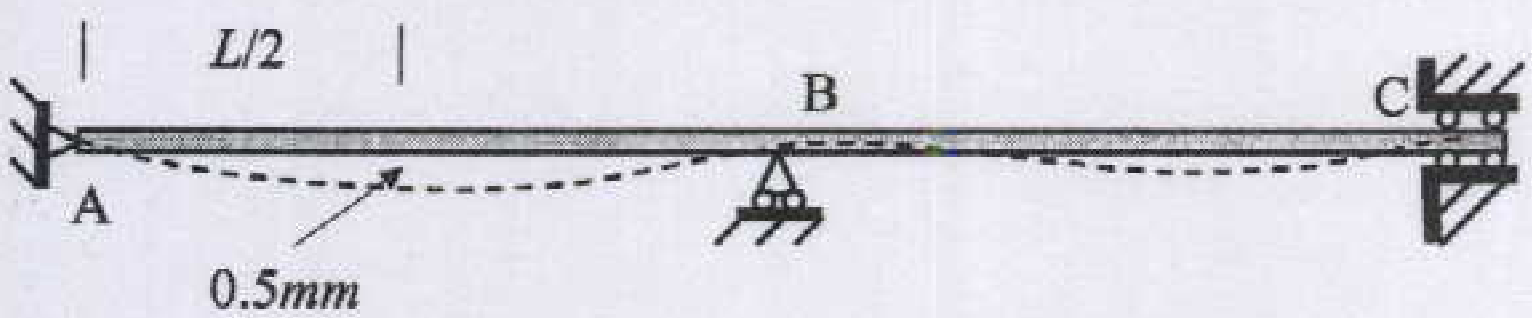
2-elements model



3-elements models



deflection due to gravity



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