UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
MIDTERM EXAMINATION

Time: 1.5 hours
Closed-book examination
One-page formula sheet allowed
Answer all **four** questions
Value of each question indicated in the bracket

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Q1. Displacements and stresses in the problem shown in Fig.1 are to be determined by using
(5) the *Rayleigh-Ritz* method, and a quadratic approximation function in the form:

$$u(x) \cong \alpha_0 + \alpha_1 x + \alpha_2 x^2 .$$

a) Transform the above approximation into the form containing only the kinematically admissible functions (i.e.: $u(x) \cong \Sigma a_i G_i(x)$, where a_i are DOFs).

b) Write the expression for the potential energy $\pi(u)$.

c) Write the equations from which the DOFs, a_i, can be calculated (do not attempt to calculate any values of DOFs!).

d) List all the detailed integrals (show explicitly the integrand and the limits) that have to be determined (do not do any integration!).

e) Explain briefly expected accuracy of the results. How that accuracy would be affected if *Galerkin*'s method were used (with the same approximation function).
Treat E, A_0, L, f_0, and **P** as given.

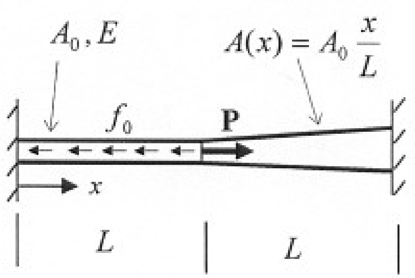


Fig. 1

Q2. Solve the problem in Fig. 1 by using *two linear* bar elements of equal length.
(10) Assume that $f_0 = \mathbf{P}/L$, and obtain the results in terms of E, A_0, L, and **P**.

a) Obtain the nodal displacements and plot the approximate solution $\tilde{u}(x)$.
b) Calculate the reactions and check the overall equilibrium.
c) Determine the element stresses.
d) Determine the nodal stresses.

It may be helpful to know the exact solution, which is:

$0 \le \bar{x} \le 1$

$$u = \frac{\mathbf{P}L}{A_0 E}\left[(c-1)\bar{x} + \frac{\bar{x}^2}{2}\right]$$

$$\sigma = \frac{\mathbf{P}}{A_0}(c - 1 + \bar{x})$$

$1 \le \bar{x} \le 2$

$$u = \frac{\mathbf{P}L}{A_0 E}(c-1)\ln\left(\frac{\bar{x}}{2}\right)$$

$$\sigma = \frac{P}{A_0}\frac{c-1}{\bar{x}}$$

where: $\bar{x} = \dfrac{x}{L}$, and $c = \dfrac{\ln 2 + 0.5}{\ln 2 + 1}$.

Q3. Use the linear bar elements and <u>symmetry</u> to analyze the truss shown.
(10) **1)** For the load as in Fig. 2. determine:

a) The displacements of node 1
b) The stresses in elements.
c) Verify equilibrium at node 1.

Any terms in the assembled matrix not needed for solving the problem may be omit

2) Additionally to force **P**, two members of the truss are subject to an increase in temperature $\Delta T = \dfrac{3}{2}\dfrac{\mathbf{P}}{AE\alpha}$ (see Fig. 3). Assume that E, α, L, A, and **P** are given. Determine the displacements of node 1, and the stress in element 1.

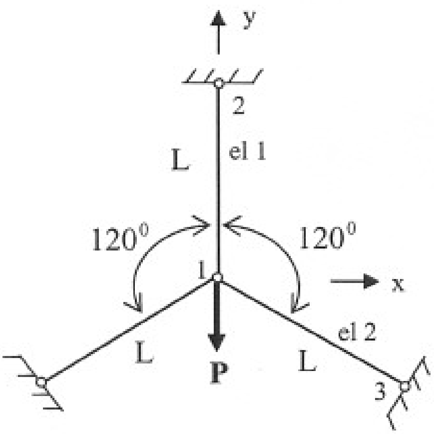


Fig. 2

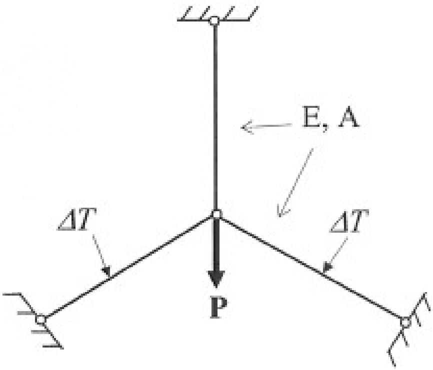


Fig. 3

Q4. Two underlined quadratic elements applied to solve *problem Q2 in A#2* render
(5) the following values of the nodal displacements:

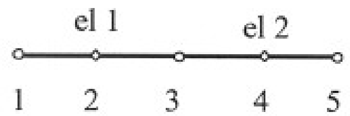
$$q_1 = 0,\ q_2 = \frac{5}{192}\frac{q_0 L^2}{AE},\ q_3 = \frac{4}{192}\frac{q_0 L^2}{AE},\ q_4 = \frac{8}{192}\frac{q_0 L^2}{AE},\ q_5 = 0$$

a) Sketch the corresponding displacement solution $\tilde{u}$ for the entire rod.

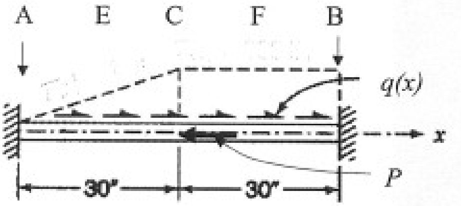
b) Determine the magnitude and location of maximum displacement $\tilde{u}$ for section AC (element 1).

c) Determine the magnitude and location of maximum displacement $\tilde{u}$ for section CB (element 2).

d) Either the results in (**b**) or the results in (**c**) should be exact. Which ones? Briefly justify your answer.



Problem Q2 in A#2



where $q(x) = \begin{cases} q_0 \dfrac{2x}{L} & \text{if } 0 \le x \le \dfrac{L}{2} \\ q_0 & \text{if } \dfrac{L}{2} \le x \le L \end{cases}$

$E = 30 \times 10^6$ psi $A = 2$ in.^{2} and L=60*in*, $q_0 = 300 \frac{lb}{in}$

$P = \frac{1}{3} q_0 L = 6{,}000\,lb$

$x_A = 0$
$x_B = 60$
$x_C = 30$
$x_E = 15$
$x_F = 45$