UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING

ME 450.3 FINITE ELEMENT ANALYSIS

MIDTERM EXAMINATION

Time: 1.5 hours
Closed-book examination
October 2006
One page formula sheet allowed
W. Szyszkowski

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Each question of equal value

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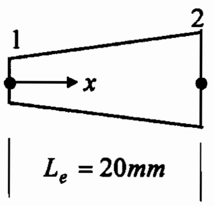
Q1. Answer questions (**a-d**) below.

a) The area of element shown is varying from $A_1 = 3mm^2$ at node 1 to $A_2 = 27mm^2$ at node 2 according to $A(x) = 3(1+2x/L_e)^2$. The volume of the element is $260mm^3$.



Determine A^e to be used in the element stiffness matrix.
If $A^e = (A_1 + A_2)/2 = 15mm^2$ were used, would the results be:

- unchanged
- less accurate
- more accurate?

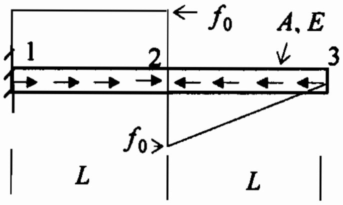
If *two* elements (each 10*mm* long) were used instead of this one element, would the nodal displacements results be:

- unchanged
- less accurate
- more accurate? Briefly explain your answers.

b) Determine the forces F_1, F_2, F_3 for the bar modeled by 3 nodes and 2 elements subjected to an increase in temperature $\Delta T = 10^0 C$ and the distributed load shown.

Given:
A=$4mm^2$, L=300*mm*, $f_0 = 2N/mm$,
$E = 2\cdot 10^5 MPa$, $\alpha = 1.25\cdot 10^{-5}/^0C$

How accurate should be the results for q_2 and q_3?

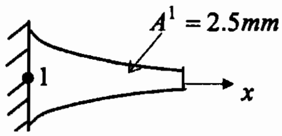


Assembled matrix:

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

c) There are no distributed load/thermal effects in element *1*.
The following has been calculated for this element at node 1:

$R_1 = -120N$ (reaction at node 1)
$\sigma^1 = 48MPa$ (element stress)
$\sigma_1^1 = 40MPa$ (nodal stress)

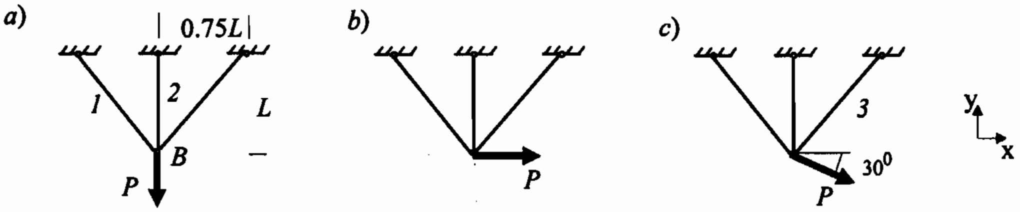


$A^1 = 2.5mm^2$ (for element *1*)

$A_1 = 3mm^2$ (at node 1)

Verify whether or not these numbers are correct.
Identify errors, if any.
Would these numbers be correct if the 'thermal' fictitious force $P_t = 60N$ were present?

d) The truss below is symmetric. The area of each member is A. The results for symmetric and anti-symmetric load P are given in figures *a* and *b* respectively. Determine the stresses in elements *1*, *2*, and *3* and the displacement of B for the load P applied as in figure *c*.



$$\sigma^1 = 0.3162\frac{P}{A}$$
$$\sigma^2 = 0.4941\frac{P}{A}$$
$$q_{Bx} = 0$$
$$q_{By} = -0.4941\frac{PL}{AE}$$

$$\sigma^1 = 0.8330\frac{P}{A}$$
$$\sigma^2 = 0$$
$$q_{Bx} = 1.7361\frac{PL}{AE}$$
$$q_{By} = 0$$

Q2. Analyze the truss shown applying *linear* bar (truss) elements. Use the numbering indicated.
You may skip calculating any terms of the assembled matrix that are not needed for solving the problem.

Assume: $A = 10mm^2$, L=200*mm*

$E = 2\cdot10^5\,MPa$, $\alpha = 1.25\cdot10^{-5}/^0C$

P=1000*N*

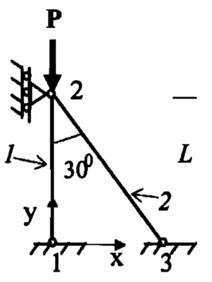


Fig. 2a

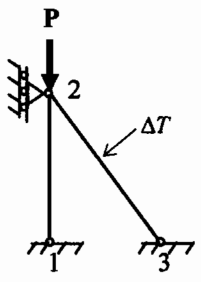


Fig. 2b

a) For the truss under load **P** (Fig. 2a) calculate:
- the displacement of node 2,
- the reaction at node 2,
- the stresses in elements *1* and *2*,
- verify equilibrium at node 2.

b) Determine the force vector for the truss under load **P** and if the temperature of element *2* increases by $\Delta T = 46.19^0C$ (Fig. 2b).
Calculate the displacement and reaction at node 2.

c) Write the ANSYS ***prep*** code (complete info on type of elements, material, geometry, boundary conditions, and loads should be included) that models the truss in Fig. 2a.

$\boxed{Q1}$ a) $A^e = \frac{1}{L_e}\int_{l_e} A(x)\,dx = \frac{V}{L_e} = \frac{260}{20} = 13\ mm^2$ 1.5

or $A^e = \frac{1}{3}(A_1 + \sqrt{A_1A_2} + A_2) = 13\ mm^2$

(2.5)

if $A_e = 15\ mm^2$ the results will be less accurate 0.5

if 2 elements are use the results will be more accurate. 0.5

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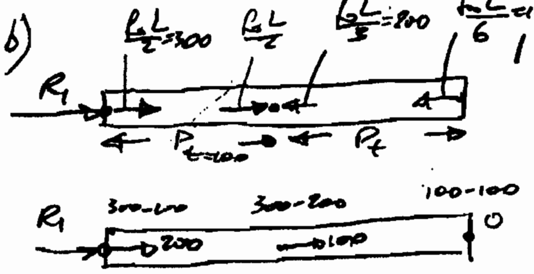
b)



1

where $f_0L = 2\cdot300 = 600$ N

$P_t = EA\alpha\Delta T = 2\cdot10^5\cdot4\cdot1.25\cdot10^{-5}\cdot10 = 100N$ 1

(2.5)

$F = \begin{bmatrix} R_1+200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+\frac{f_0L}{2}-P_t \\ \frac{f_0L}{6} \\ -\frac{f_0L}{6}+P_t \end{bmatrix}$

q_3, q_3 would be exact. 0.5

---

c)

(2.5)

Check: $P_1^e = R_1 = -\sigma_1' A_1 = \frac{A^{①}E}{L_e}(q_1-q_2) = -A^{①}\sigma^{①}$

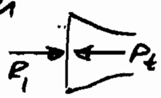
$-120 = -40\cdot3$ ✓ 1

and $\sigma_1' = \frac{A^{①}}{A_1}\sigma^{①}$

$40 = \frac{2.5}{3}\cdot48$ ✓ 1

if P_t were present

then



and the following applies:

$R_1 - P_t = -\sigma_1' A_1 = -A^{①}\sigma^{①}$ 0.5

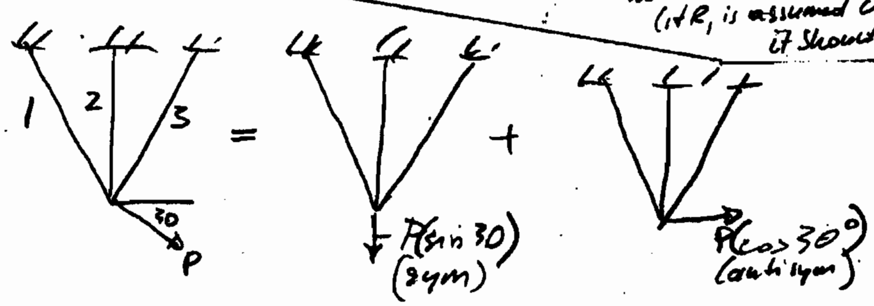
$-120-60 = -120 = -120$

not correct.

(if R_1 is assumed correct then it should be $\sigma_1' = 80, \sigma^1 = 72$)

---

d)



(2.5)

$\sigma^1 = 0.3162\cdot\sin30 + 0.8330\cdot\cos30° = 0.8795\frac{P}{A}$ 0.5

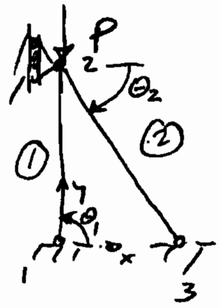
$\sigma^2 = 0.9941\sin30 + 0 = 0.297\frac{P}{A}$ 0.5

$\sigma^3 = 0.3162\cdot\sin30 - 0.8330\cdot\cos30° = -0.5633\frac{P}{A}$. 0.5

$q_{Bx} = 0 + 1.7361\cdot\cos30° = 1.5035\frac{PL}{AE}$ 0.5

$q_{By} = -0.4941\sin30° = -0.2470\frac{PL}{AE}$. 0.5

Q2



$\ell = \cos\theta$
$m = \sin\theta$

el.	nodes	θ	ℓ	m	L_e
1	1-2	90	0	1	$L=200$
2	2-3	$-60°$	0.5	$-\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}L=230.94$

Only $q_{2y} \neq 0$.

$$K^{①} = \frac{AE}{L}\begin{array}{c|c} \begin{array}{cc} 1 & \downarrow q_{2y} \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} & \begin{array}{l} 1 \\ \\ q_{2x} \\ \leftarrow q_{2y} \end{array} \end{array}$$

②

$$K^{②} = \frac{AE}{L}\frac{\sqrt{3}}{2}\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix} = \frac{AE}{L}\begin{array}{c|c} \overset{q_{2y}}{\downarrow} & \\ \begin{bmatrix} \cdot & -\frac{3}{8} & \cdot & \cdot \\ \cdot & \frac{3\sqrt{3}}{8} & \cdot & \cdot \\ \cdot & \frac{3}{8} & \cdot & \cdot \\ \cdot & -\frac{3\sqrt{3}}{8} & \cdot & \cdot \end{bmatrix} & \begin{array}{l} 2 \\ \\ 3 \\ \\ \end{array} \end{array}$$

a) Assembly

$$\boxed{\frac{AE}{L}}\begin{bmatrix} q_{1x} & q_{1y} & q_{2x} & q_{2y} & q_{3x} & q_{3y} \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\frac{3}{8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \boxed{1+\frac{3\sqrt{3}}{8}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{3}{8} & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\frac{\sqrt{3}\cdot 3}{8} & \cdot & \cdot \end{bmatrix}\begin{bmatrix} q_{1x}=0 \\ q_{1y}=0 \\ q_{2x}=0 \\ q_{2y} \\ q_{3x}=0 \\ q_{3y}=0 \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} \\ R_{2x} \\ -P \\ R_{3x} \\ R_{3y} \end{bmatrix}$$

1.6495

$\frac{10\cdot 2\cdot 10^5}{200} = 10^4$

$R_{2x} = \frac{AE}{L}\left(-\frac{3}{8}\right)q_{2y}$

$$\rightarrow \frac{AE}{L}\left(1+\frac{3\sqrt{3}}{8}\right)q_{2y} = -P$$

displ at 2

$$q_{2y} = -\underbrace{\frac{1}{1+\frac{3\sqrt{3}}{8}}}_{0.6062}\ \underbrace{\frac{PL}{AE}}_{0.1} = -0.060624\text{ mm}$$ ①

reaction at 2

$$+\overset{0.375}{\frac{3}{8}}\frac{AE}{L}\cdot\frac{1}{1+\frac{3\sqrt{3}}{8}}\frac{PL}{AE} = \overset{0.2273}{\frac{3}{8+3\sqrt{3}}}P = \underline{227.3\,N = R_{2y}}$$

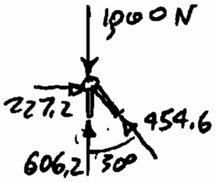
stresses

$$\sigma^{①} = \frac{E}{L}\left(\overset{m(q_{2y}-q_{1y})}{-0.6062\frac{PL}{AE}}\right) = -0.6062\frac{P}{A} = -60.62\text{ MPa}$$

①

$$\sigma^{②} = \frac{E}{\frac{2}{\sqrt{3}}L}\left(\overset{m(q_{3y}-q_{2y})}{-\frac{\sqrt{3}}{2}\cdot 0.6062\frac{PL}{AE}}\right) = -0.4546\frac{P}{A} = -45.96\text{ MPa}$$

node 2

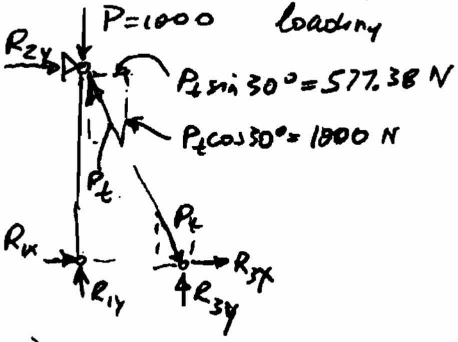


①

equilibrium Ok.

b) if $\Delta T = 96.19°$ for el 2

$P_t = AE\alpha\Delta T = 10\cdot 2\cdot 10^5\cdot 1.75\cdot 10^{-5}\cdot 96.19° = 1{,}159.75\,N$



$$F = \begin{bmatrix} R_{1x} \\ R_{1y} \\ R_{2x}-577.38 \\ 0 \\ R_{3x}+577.38 \\ R_{3y}-1000 \end{bmatrix} \rightarrow$$

F_{2y}

①

then $\underline{q_{2y}=0.}$ ①

and $R_{2y}-577.38=0$

① $\underline{R_{2y} = 577.38\,N}$

c)

```
/prep7
et,1,link1
mp,ex,1,2e5
r,1,10

n,1
n,2,0,200
n,3,115.97

e,1,2
e,2,3

d,1,all
d,3,all
d,2,ux

f,2,fx,-1000
fini
```

②