UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING

# ME 450.3 FINITE ELEMENT ANALYSIS

## MIDTERM EXAMINATION

Time: 1.5 hours
Closed-book examination
One page formula sheet allowed

October 2006
W. Szyszkowski

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Each question of equal value

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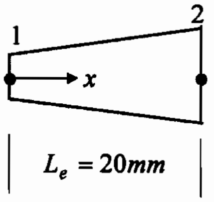
**Q1**. Answer questions (**a-d**) below.

**a**) The area of element shown is varying from $A_1 = 3mm^2$ at node 1 to $A_2 = 27mm^2$ at node 2 according to $A(x) = 3(1 + 2x/L_e)^2$. The volume of the element is $260mm^3$.



Determine $A^e$ to be used in the element stiffness matrix.

If $A^e = (A_1 + A_2)/2 = 15mm^2$ were used, would the results be:

- unchanged
- less accurate
- more accurate?

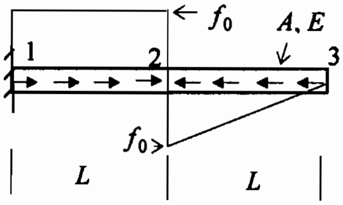
If *two* elements (each 10*mm* long) were used instead of this one element, would the nodal displacements results be:

- unchanged
- less accurate
- more accurate? Briefly explain your answers.

**b**) Determine the forces $F_1, F_2, F_3$ for the bar modeled by 3 nodes and 2 elements subjected to an increase in temperature $\Delta T = 10^0 C$ and the distributed load shown.

Given:
$A$=$4mm^2$, $L$=300*mm*, $f_0 = 2N/mm$,
$E = 2 \cdot 10^5 MPa$, $\alpha = 1.25 \cdot 10^{-5}/^0C$

How accurate should the results be for $q_2$ and $q_3$?

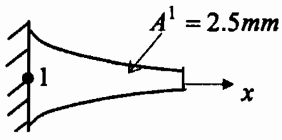


Assembled matrix:

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

**c**) There are no distributed load/thermal effects in element *1*.
The following has been calculated for this element at node 1:

$R_1 = -120N$ (reaction at node 1)
$\sigma^1 = 48MPa$ (element stress)
$\sigma_1^1 = 40MPa$ (nodal stress)



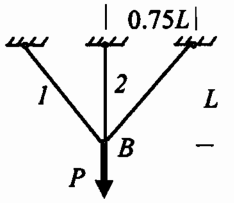
$A^1 = 2.5mm^2$ (for element *1*)

$A_1 = 3mm^2$ (at node 1)

Verify whether or not these numbers are correct.
Identify errors, if any.
Would these numbers be correct if the 'thermal' fictitious force $P_t = 60N$ were present?

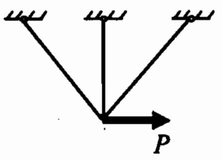
**d)** The truss below is symmetric. The area of each member is $A$. The results for symmetric and anti-symmetric load $P$ are given in figures *a* and *b* respectively. Determine the stresses in elements *1*, *2*, and *3* and the displacement of *B* for the load $P$ applied as in figure *c*.

*a)*



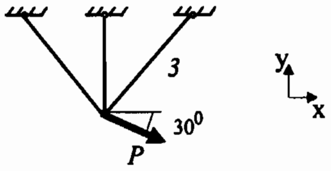
$\sigma^1 = 0.3162\frac{P}{A}$
$\sigma^2 = 0.4941\frac{P}{A}$
$q_{Bx} = 0$
$q_{By} = -0.4941\frac{PL}{AE}$

*b)*



$\sigma^1 = 0.8330\frac{P}{A}$
$\sigma^2 = 0$
$q_{Bx} = 1.7361\frac{PL}{AE}$
$q_{By} = 0$

*c)*



**Q2.** Analyze the truss shown applying *linear* bar (truss) elements. Use the numbering indicated. You may skip calculating any terms of the assembled matrix that are not needed for solving the problem.

Assume: $A = 10mm^2$, $L$=200*mm*
$E = 2 \cdot 10^5 MPa$, $\alpha = 1.25 \cdot 10^{-5}/^0C$
P=1000*N*

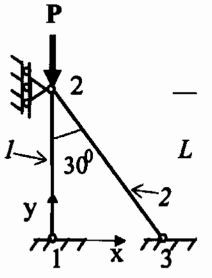


Fig. 2a

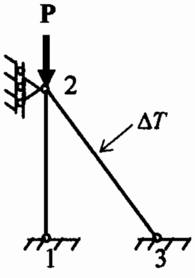


Fig. 2b

**a)** For the truss under load **P** (Fig. 2a) calculate:
- the displacement of node 2,
- the reaction at node 2,
- the stresses in elements *1* and *2*,
- verify equilibrium at node 2.

**b)** Determine the force vector for the truss under load **P** and if the temperature of element *2* increases by $\Delta T = 46.19^0C$ (Fig. 2b).
Calculate the displacement and reaction at node 2.

**c)** Write the ANSYS ***prep*** code (complete info on type of elements, material, geometry, boundary conditions, and loads should be included) that models the truss in Fig. 2a.

$\boxed{Q1}$ a) $A^e = \frac{1}{L_e}\int_{L_e} A(x)dx = \frac{V}{L_e} = \frac{260}{20} = 13\ mm^2$ 1.5

(2.5)

or $A^e = \frac{1}{3}(A_1+\sqrt{A_1A_2}+A_2) = 13 mm^2$

if $A_e = 15 mm^2$ the results will be less accurate 0.5

if 2 elements are use the results will be more accurate. 0.5

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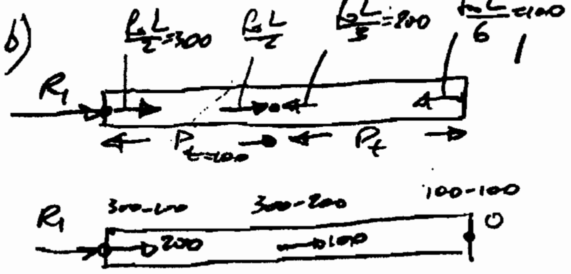
b)

(2.5)



Where $f_oL = 2\cdot300 = 600N$

$P_t = EA\alpha\Delta T = 2\cdot10^5\cdot4\cdot1.25\cdot10^{-5}\cdot10 = 100N$ 1

$$F = \begin{bmatrix} R_1+200\\ 100\\ 0\end{bmatrix} = \begin{bmatrix} R_1+\frac{f_oL}{2}-P_t\\ \frac{f_oL}{6}\\ -\frac{f_oL}{6}+P_t\end{bmatrix}$$

$q_3, q_3$ would be exact. 0.5

---

c)

(2.5)

Check: $P_1^e = R_1 = -\sigma_1' A_1 = \frac{A^{(1)}E}{L_e}(q_1-q_2) = -A^{(1)}\sigma^{(1)}$

$-120 = -40\cdot3$ ✓ 1

and $\sigma_1' = \frac{A^{(1)}}{A_1}\sigma^{(1)}$

$40 = \frac{2.5}{3}\cdot48$ ✓ 1

if $P_t$ were present. then

and the following applies:

$R_1 - P_t = -\sigma_1' A_1 = -A^{(1)}\sigma^{(1)}$ 0.5

$-120-60 = -120 = -120$

not correct.

(if $R_1$ is assumed correct then it should be $\sigma_1'=80$, $\sigma^1=72$)

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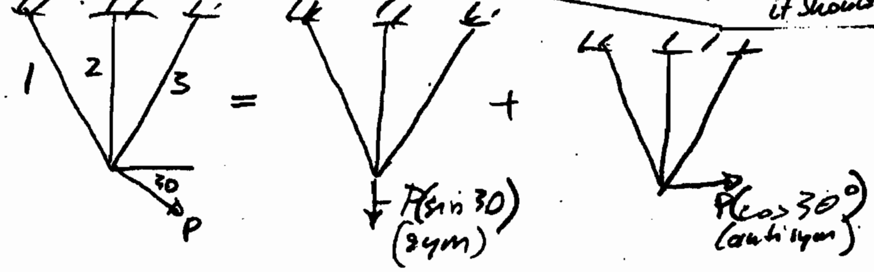
d)

(2.5)



$\sigma^1 = 0.3162\cdot\sin30 + 0.8330\cdot\cos30° = 0.8795\frac{P}{A}$ 0.5

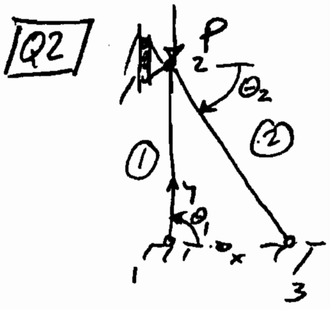
$\sigma^2 = 0.9941\sin30 + 0 = 0.297\frac{P}{A}$ 0.5

$\sigma^3 = 0.3162\cdot\sin30 - 0.8330\cdot\cos30° = -0.5633\frac{P}{A}$. 0.5

$q_{Bx} = 0 + 1.7361\cdot\cos30° = 1.5035\frac{PL}{AE}$ 0.5

$q_{By} = -0.9941\sin30° = -0.2970\frac{PL}{AE}$. 0.5

## Q2



Only $q_{2y} \neq 0$.

| el. | nodes | $\theta$ | $l$ | $m$ | $L_e$ |
|---|---|---|---|---|---|
| 1 | 1-2 | 90 | 0 | 1 | $L=200$ |
| 2 | 2-3 | $-60°$ | 0.5 | $-\frac{\sqrt{3}}{2}$ | $\frac{2}{\sqrt{3}}L=230.94$ |

$l=\cos\theta$
$m=\sin\theta$

$$K^{①}=\frac{AE}{L}\begin{array}{c}\overbrace{\hphantom{0\quad 0}}^{1}\quad \downarrow q_{2y}\\ \left[\begin{array}{cccc}0&0&0&0\\0&1&0&-1\\0&0&0&0\\0&-1&0&1\end{array}\right]\end{array}\begin{array}{l}\\ \big\}\,1\\ \\ q_{2x}\\ \leftarrow q_{2y}\end{array}$$

②

$$K^{②}=\frac{AE}{L}\frac{\sqrt{3}}{2}\begin{array}{c}\overbrace{\hphantom{\frac{1}{4}\quad -\frac{\sqrt{3}}{4}}}^{2}\ \overbrace{\hphantom{-\frac{1}{4}\quad \frac{\sqrt{3}}{4}}}^{3}\\ \left[\begin{array}{cccc}\frac{1}{4}&-\frac{\sqrt{3}}{4}&-\frac{1}{4}&\frac{\sqrt{3}}{4}\\-\frac{\sqrt{3}}{4}&\frac{3}{4}&\frac{\sqrt{3}}{4}&-\frac{3}{4}\\-\frac{1}{4}&\frac{\sqrt{3}}{4}&\frac{1}{4}&-\frac{\sqrt{3}}{4}\\\frac{\sqrt{3}}{4}&-\frac{3}{4}&-\frac{\sqrt{3}}{4}&\frac{3}{4}\end{array}\right]\end{array}=\frac{AE}{L}\begin{array}{c}\quad q_{2y}\downarrow\\ \left[\begin{array}{cccc}\cdot&-\frac{3}{8}&\cdot&\cdot\\\cdot&\frac{3\sqrt{3}}{8}&\cdot&\cdot\\\cdot&\frac{3}{8}&\cdot&\cdot\\\cdot&-\frac{3\sqrt{3}}{8}&\cdot&\cdot\end{array}\right]\end{array}\begin{array}{l}\big\}\,2\\ \\ \big\}\,3\end{array}$$

a) Assembly

$$\boxed{\frac{AE}{L}}\left[\begin{array}{cccccc}q_{1x}&q_{1y}&q_{2x}&q_{2y}&q_{3x}&q_{3y}\\ &&&0&&\\&&&-1&&\\&&&-\frac{3}{8}&&\\&&&\boxed{1+\frac{3\sqrt{3}}{8}}&&\\&&&\frac{3}{8}&&\\&&&-\frac{\sqrt{3}\cdot 3}{8}&&\end{array}\right]\begin{bmatrix}q_{1x}=0\\q_{1y}=0\\q_{2x}=0\\q_{2y}\\q_{3x}=0\\q_{3y}=0\end{bmatrix}=\begin{bmatrix}R_{1x}\\R_{1y}\\R_{2x}\\-P\\R_{3x}\\R_{3y}\end{bmatrix}$$

1.6495

$\frac{10\cdot 2\cdot 10^5}{200}=10^4$

$R_{2x}=\frac{AE}{L}\left(-\frac{3}{8}\right)q_{2y}$

$\rightarrow \frac{AE}{L}\left(1+\frac{3\sqrt{3}}{8}\right)q_{2y}=-P$

disp at 2

$$q_{2y}=-\underbrace{\frac{1}{1+\frac{3\sqrt{3}}{8}}}_{0.6062}\ \underbrace{\frac{PL}{AE}}_{0.1}=-0.060624\text{ mm}$$

①

reaction at 2

$$+\overset{0.375}{\frac{3}{8}}\frac{AE}{L}\cdot\frac{1}{1+\frac{3\sqrt{3}}{8}}\frac{PL}{AE}=\overset{0.2273}{\frac{3}{8+3\sqrt{3}}}P=\underline{227.3\text{ N}=R_{2y}}$$

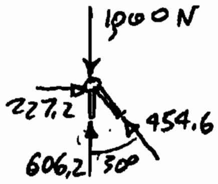
stresses

$$\sigma^{①}=\frac{E}{L}\Big(\overset{m(q_{2y}-q_{1y})}{-0.6062\frac{PL}{AE}}\Big)=-0.6062\frac{P}{A}=-60.62\text{ MPa}$$

①

$$\sigma^{②}=\frac{E}{\frac{2}{\sqrt{3}}L}\Big(\overset{m(q_{3y}-q_{2y})}{-\frac{\sqrt{3}}{2}\cdot 0.6062\frac{PL}{AE}}\Big)=-0.4546\frac{P}{A}=-45.96\text{ MPa}$$

node 2

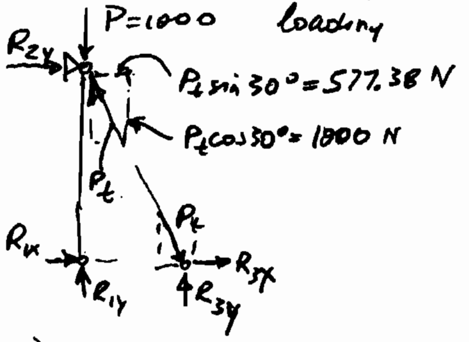


①

equilibrium Ok.

b) if $\Delta T=96.19°$ for el 2

$P_t=AE\alpha\Delta T=10\cdot 2\cdot 10^5\cdot 1.75\cdot 10^{-5}\cdot 96.19°=1,159.75\text{ N}$



$$F=\begin{bmatrix}R_{1x}\\R_{1y}\\R_{2x}-577.38\\0\\R_{3x}+577.38\\R_{3y}-1000\end{bmatrix}\rightarrow$$

$F_{2y}$

①

then $\underline{q_{2y}=0.}$ ①

and $R_{2y}-577.38=0$

① $\underline{R_{2y}=577.38\text{ N}}$

c)

```
/prep7
et,1,link1
mp,ex,1,2e5
r,1,10
```

```
n,1
n,2,0,200
n,3,115.97
```

```
e,1,2
e,2,3
```

```
d,1,all
d,3,all
d,2,ux
```

```
f,2,fx,-1000
fini
```

②