

UNIVERSITY OF SASKATCHEWAN
DEPARTMENT OF MECHANICAL ENGINEERING
ME 450.3 FINITE ELEMENT ANALYSIS
MIDTERM EXAMINATION

Time: 1.5 hours
Closed-book examination
One-page formula sheet allowed
Each question of equal value

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Q1. You are to analyze the bar shown loaded by uniformly distributed load f_0 and force P applied at C.

a) Use the R-R method and the displacements assumed as $\tilde{u}(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$.

Identify the kinematically admissible trial functions $G_i(x)$, and the corresponding DOFs.

Write the expression for the work potential $W(\tilde{u})$ and then write the R-R equations.

Solve for the DOFs.

Calculate the displacements of points C and D.

Calculate the stresses at points C and D.

Plot $\tilde{u}(x)$ and $\tilde{\sigma}(x)$, compare with the exact solution.

b) Use two linear bar element (and nodes 1, 2, 3 as shown).

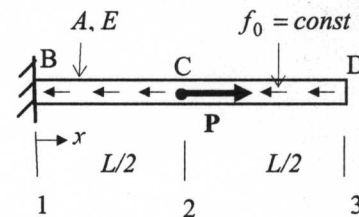
Clearly identify the nodal forces.

Determine the nodal displacements and plot \tilde{u} .

Determine the element stresses and plot $\tilde{\sigma}_{elem}$.

Determine the nodal stresses and plot $\tilde{\sigma}_{nodal}$.

Compare with the exact solution



Given: $f_0 = 5 \text{ N/mm}$, $P = f_0 L = 2,000 \text{ N}$

$E = 200 \cdot 10^3 \text{ MPa}$, $A = 10 \text{ mm}^2$, $L = 400 \text{ mm}$

Hint: Obtain the solutions in terms of f_0, L, A, E first, and then substitute the numbers.

The exact solution is given by:

$$u_{exact} = 0.2\bar{x}^2 \text{ mm} \text{ if } 0 \leq \bar{x} \leq 0.5 \text{ and } u_{exact} = 0.2(1-\bar{x})^2 \text{ mm} \text{ if } 0.5 \leq \bar{x} \leq 1.0 \quad \text{where } \bar{x} = x/L$$

$$\sigma_{exact} = 200\bar{x} \text{ MPa} \text{ if } 0 \leq \bar{x} < 0.5 \text{ and } \sigma_{exact} = -200(1-\bar{x}) \text{ MPa} \text{ if } 0.5 < \bar{x} \leq 1.0$$

Q2. Analyze the truss shown by applying linear bar (truss) elements. Use the numbering indicated.

All elements have the same length and area.

Force P is applied to node 1 and temperature of elements e2 and e3

increases by $\Delta T = 30^\circ \text{ C}$ (temperature of e1 remains unchanged)

a) Determine:

- the displacement of node 1,
- the stresses in the elements,
- verify equilibrium of node 1.

You may calculate any terms of the elements and

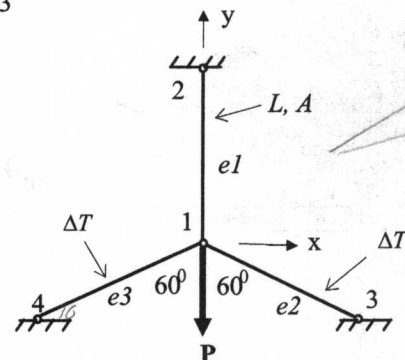
assembled matrices that are not needed for solving the problem

In particular, pay attention to terms related to node 1.

Also, notice the symmetry.

Assume: $P = 1,500 \text{ N}$, $L = 400 \text{ mm}$, $A = 10 \text{ mm}^2$

$E = 200 \cdot 10^3 \text{ MPa}$, $\alpha = 1.25 \cdot 10^{-5} / ^\circ \text{ C}$



b) Write a part of the ANSYS *prep* code that contains the complete info on the type of elements, material, geometry, and boundary conditions for the truss.