MATH 224.3 Mid-Term Test 2 Student Name [redacted] Number [redacted]

Simply write down your answers in the spaces provided.

The questions are worth 2 marks each.

1. Evaluate the limit of the sequence { n sin (4/n) }.

$\lim_{n\to\infty} n \sin(4/n) \to \cancel{(\infty)(0)}$

$\lim_{n\to\infty} \frac{\sin 4n^{-1}}{n^{-1}} = \lim_{n\to\infty} \frac{\cos(4/n)(-1)4n^{-2}}{-1n^{-2}} = \cancel{\lim_{n\to\infty} \cos(4/n)\, 4}$

$\boxed{= 4}$

2

In questions 2. to 4. state whether the series converges or diverges and give your reasons.

2. $\sum_{n=0}^{\infty} 4^n/3^{2n}$

$\lim_{n\to\infty} \frac{4^n}{3^{2n}} = 0$

converges

limit goes to 0

2n goes to infinity faster than n

$\frac{4^n}{(3^2)^n} = \left(\frac{4}{9}\right)^n$

$r = \frac{4}{9}, |r| < 1;$

1 1/2

3. $\sum_{n=1}^{\infty} ne^{-n}$

$\cancel{\lim_{n\to\infty} \frac{1n}{1e^n} = \lim_{n\to\infty} \frac{1}{e^n}} = 0$

2

limit ~~goes~~ to 0

converges

4. $\sum_{n=1}^{\infty} \text{Cos}^{-1}(1/n)$

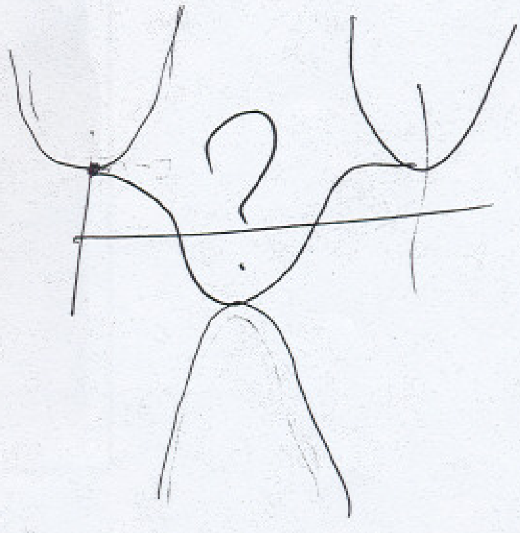
$\lim_{n\to\infty} \cos^{-1}(1/n)$

$= \cancel{\cos^{-1}(0)}$

$= 1$

diverge not 0

2



$$\frac{15}{16}$$

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5. Find the interval of convergence for the series $\sum_{n=0}^{\infty} (x-2)^n / 4^n$

$-R \le x - c \le R$

$a_n(x-c)^n$ $a_n = \frac{1}{4^n}$

$c = 2$

$$R = \lim_{n \to \infty} \frac{1}{\left|\frac{a_{n+1}}{a_n}\right|} = \lim_{n \to \infty} \frac{1}{|a_n|^{1/n}}$$

$$\lim_{n \to \infty} \frac{1}{\left|\frac{1}{4^n}\right|^{1/n}} = \lim_{n \to \infty} \frac{1}{\left(\frac{1}{4}\right)} = 4 = R$$

$-4 \le x - 2 \le 4$

2

$$\boxed{-2 \le x \le 6}$$

6. Find the power series expansion of $f(x) = (1 + x^2)^{1/2}$ about $x = 0$.

n		
0	$f(x) = (1+x^2)^{1/2}$ $f(0) = 1$	
1	$f'(x) = \frac{1}{2}(1+x^2)^{-1/2}(2x) = x(1+x^2)^{-1/2}$	$f'(0) = 0$
2	$f''(x) = x\left(-\frac{1}{2}(1+x^2)^{-3/2}(2x)\right) + (1+x^2)^{-1/2}(1)$ $= -x^2(1+x^2)^{-3/2} + (1+x^2)^{-1/2}$	$f''(0) = 0 + 1 = 1$
3	$f'''(x) = -x^2\left(\frac{-3}{2}(1+x^2)^{-5/2} 2x\right) + (1+x^2)^{-3/2}(-2x) + \left(\frac{-1}{2}\right)(1+x^2)^{-3/2} 2x$	

$a_n(x-c)^n$

$$\text{Taylor} = \frac{f^n(c)(x-c)^n}{n!}$$

2

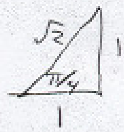
$= 3x^3(1+x^2)^{-5/2} - 3x(1+x^2)^{-3/2}$ $f'''(0) = 0$

$4) = 3x^3\left(\frac{-5}{2}\right)(1+x^2)^{-7/2} 2x + (1+x^2)^{-5/2} 9x^2 - 3x\left(\frac{-3}{2}\right)(1+x^2)^{-5/2}(2x) - 3(1+x^2)^{-3/2}$ $f^{iv}(0) = -3$

$$f(x) = 1 + \frac{x^2}{2!} - \frac{3x^4}{4!} \ldots$$

Σ ?

7. Find the power series expansion of f(x) = sin x about x = π/4.



n		
0	$f(x) = \sin x$	$f\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
1	$f'(x) = \cos x$	$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$
2	$f''(x) = -\sin x$	$f''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
3	$f'''(x) = -\cos x$	$f'''\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
4	$f^{IV}(x) = \sin x$	$f^{IV}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(c)(x-c)^{n}}{n!}$$

$$f(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^2}{2!} - \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^3}{3!} + \frac{\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^4}{4!} \ldots$$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{+1^{n}(-1)^{(2(n+1))}\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)^{n}}{n!}}$$

2

8. How many terms in the Maclaurin series for f(x) = exp(−x) guarantee a truncation error of less than 10^{-5} for all x in the interval 0 ≤ x ≤ 2 ? [Simply give an inequality for n, no need to calculate a value for the number n.]

$f(x) = e^{-x}$

amount of n

$R_n < 10^{-5}$

remainder

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \frac{f^{n+1}(z_n)(x-c)^{n+1}}{(n+1)!}$$

$$R_n = \frac{f^{n+1} z_n (x-c)^{n+1}}{n+1}$$

$f^{(n+1)}(z_n) = (-1)^{(n+1)} e^{-z_n}$

u n: 0

e^{-x}

f' $-e^{-x}$

f^2

$$\boxed{\frac{(-1)^{(n+1)} e^{-z_n} (x-2)^{n+1}}{(n+1)!} < 10^{-5}}$$

$1\frac{1}{2}$